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Humanism and Modern Mathematics

DAVID MALCOLM

INTRODUCTION

It would appear at first glance that mathematics is a subject which is immune from religious biases, because it seeks pure abstract truth. Nevertheless such is far from the case. The religious presuppositions of mathematicians have indeed influenced their mathematical work. We want to look particularly at Georg Cantor, who did most of his work in the last part of the nineteenth century, because he has had a tremendous influence on the direction of twentieth century mathematics.

This century, one of the main areas exercising mathematicians is the very foundation of mathematics, in set theory. I believe that this work is misdirected, and as a consequence mathematics is being robbed of its foundations. To the mathematicians, I would ask that you bear with me, as we consider the direction of modern mathematics. Perhaps some of the views expressed here are at odds with the majority view, but the question is: Does modern mathematics have a solid foundation in the divine revelation which we have in the Bible?

BASIS OF MATHEMATICS

What is the philosophical basis for mathematics? We may get many different answers to that question, depending upon the religious or philosophical position of the person answering.

To the humanist, who sees man as the ultimate intelligence, the answer will be along the lines that man is master of all things, existing and imaginary; therefore man will naturally want to rationalise his world so he can understand it. Indeed we are quite within our rights to investigate mathematical universes of our own construction.

On the other hand, if we believe that the Bible is a divine revelation from the One who created this universe, we will see things differently. We will see man as the prince of creation, as he submits to his Creator's will. I make no apology for the fact that I take this as my starting point. God made this universe. He made man to have dominion over this creation. And furthermore He made man in His own image (Psalm 8, and Genesis 1:26). Therefore man is equipped with the ability to correctly understand and administer this creation; although we may not be able to comprehend that which is outside this creation. God has certainly not allowed us to completely comprehend Him, in His infinite majesty.

On this basis it may be that there will be things we cannot completely understand within this creation, for which we must trust the Lord. 'Trust in the Lord with all your heart, And do not lean on your own understanding" (Proverbs 3:5 — All scripture quotes are from the NASB). "As he [the mathematics student] strains his mental faculties to their utmost, he will realize that there are many truths — in the physical as well as the spiritual realm — that are simply beyond his ability to understand."[1] For example π, the ratio of the circumference of a circle to its diameter. We often use the rational approximation 22/7, so school students can cancel out, but this approximation is only accurate to 2 decimal places. In 1949, one of the early computers ENIAC was used to calculate π to 2037 places, then in 1967, a CDC 6600 was used to obtain an approximation correct to 500,000 places;[2] but man will never be able to say he knows the value of π.

CANTOR'S CONTRIBUTIONS

Georg Cantor has contributed humanist thinking to mathematics in several areas. Not that he is the only one to suggest these lines of reasoning, nor perhaps the first to do so, but he seems to have contributed more than his fair share. Most importantly his thoughts have become accepted, and exert a considerable influence on modern mathematical thinking. There are three areas of concern to us:

1. MATHEMATICS IS A GAME. Cantor's famous aphorism says: "The essence of mathematics lies in its freedom."[3] Other people have said the same thing even more strongly. Hilbert said: "Mathematics is nothing more than a game played according to certain simple rules with meaningless marks on paper."[4] This is certainly not how most of us see it. Mathematics is surely vital to man's God-given function of exercising dominion over this creation. What science could there be without mathematics? We would not be able to measure anything. How could there be any trade without basic mathematics? Where would technology be? Henry Ford would never have been able to make interchangeable components for his production lines. Whatever else can be said about mathematics, surely it has its basis in reality.

2. TRANSFINITE NUMBERS. Cantor developed a theory which presents a formalised treatment of infinites,[5] and utilized the concept of "denumerability". He was fully aware that he was opposing the teaching of great mathematicians of the past in taking this position.[6] But surely mathematics should only embody concepts we can understand, and God has clearly not allowed us to comprehend infinity. "The heavens are the heavens of the Lord; but the earth he has given to the sons of men" (Psalm 115:16). Bell clearly understands this verse of scripture as I do: his chapter on the formalised treatment of infinity is entitled "Storming the heavens".[7] God is infinite and inhabits eternity, but such realities we understand only as "in a mirror dimly" at the present time (1 Corinthians 13:12). To actually formalise a treatment of the infinite, would therefore seem to be a case of man exalting himself to a position of omniscience.

3. PROOF OF NUMBER THEORY. Cantor was one of the main influences behind the development of set theory. The main thing about his Set Theory was that from it he provided a proof of the basic axioms of arithmetic, or at least he thought he did. Actually his proof raised problems which have still not been resolved, in spite of 80 years of intensive work by many brilliant men. It has led to a serious questioning of the very basis of the whole of mathematics, which we shall look at. But the point is: is a proof needed for the elements of arithmetic? I suggest that a sufficient proof lies in the fact that all mankind understands it. Even my five year old daughter when starting school was able to do some simple divisions correctly. It would seem that these basic concepts have been placed in us by God, for our task of administering this creation. We note also that throughout the scriptures God communicates with man in number concepts, showing that the parties correctly understand each other.

One author has written that Cantor's father was a Jew converted to Protestantism, while his mother was born a Catholic; and that he was deeply religious,[8] but whatever his religion was, I would question whether it was founded on the God of the scriptures. Cantor seems to have had a great respect for medieval theological contemplation of the infinite. This may explain why he worked along the lines he did.

MATHEMATICS IS RELATED TO REALITY

It is not a recent idea to view mathematics as a game. Geometry as developed by the ancient Greeks seems to have been something of a game indulged in by the "educated" rich.[9] The tools allowed in this "game" were a straightedge (not graduated), and a compass which was not able to be preset. (That is, you could draw a circle with a given centre, through a given point, but not with a given radius). If it was not a game, why did such artificial rules apply?

Most of the great mathematicians of the past acknowledged that mathematics is related to the real world: Fourier "expressed very definitely his view that mathematics only justifies itself by the help it gives towards the solution of physical problems";[10] and stated further: "The deep study of nature is the most fruitful source of mathematical discovery."[11] Gauss took a motto from King Lear: "Thou, nature, art my goddess; to thy laws my services are bound" meaning that Gauss believed that mathematics must touch the real world.[12] It is clear from what Brouwer said in his inaugural address at the University of Amsterdam in 1912, that he believed that mathematics is the basis of science which in turn is the means whereby we understand nature.[13] May I presume to say that these views are correct. My background is in engineering; and in my university course, we were only taught branches of mathematics applicable to the solution of physical problems. "It is proper to tie mathematics, though an expression of the human mind, with the physical universe; for both . . . are the work of the same Creator."[14]

Not that mathematicians necessarily have to be aware of the application of the work at the time of development. Newton was asked in August 1648: "What is the path of a body attracted by a force directed toward a fixed point, the force varying in intensity as the inverse square of the distance?" Newton was able to instantly answer "an ellipse" because he had previously been investigating this matter mathematically.[15] The question was asked by the astronomer Edmund Halley, and thus originated the "Principia", Newton's significant work. The mathematics had been done before the physical usefulness was seen. Also Riemann did a lot of work on non-Euclidian geometry, without perhaps perceiving any application for it at the time; but the new branch of mathematics he devised now seems to apply to real astronomical space, which is thought to be curved by the presence of matter.[16]

The point is, that the final test of mathematics lies in its relevance to the real world, not in how pleasing it may be to our intellect.

APPROACHING INFINITY

Mathematics has been concerned with the infinite for quite some time. The Greek mathematicians formulated a construction which seemed to show that an athlete (Achilles) can never catch up to a tortoise, if the tortoise is given a head start. This is known as one of Zeno's paradoxes.[17] The argument goes as follows: suppose the tortoise is given a 100 metres start, and that Achilles can run ten times as fast as the tortoise, then in the time it takes Achilles to cover that 100 metres, the tortoise will have advanced by 10 metres; in the time it takes Achilles to cover that 10 metres the tortoise will cover a further metre, and so on. For any head start the tortoise has, while Achilles is covering that distance the tortoise will advance a tenth of the distance. Thus it can be seen that we have constructed an infinite series, and only when we have progressed through every term, can Achilles catch the tortoise. The Greeks gave up on the problem, and kept well away from such troublesome areas of mathematics.

Why didn't the Greek mathematicians resolve the paradox? Why did it need 2000 years before it was resolved? Perhaps the answer is simple: their religious bias suggested that the gods were arbitrary, and didn't really have control over the universe anyway. Why should they have expected the world to be consistent, and free from contradictions? It was when scientists and mathematicians began to take the Bible seriously, that they would really have expected the universe to be a unity.

The solution of the paradox is simple (to someone with a good modern education in mathematics). The infinite series (for the distance the tortoise travels before being caught) has a finite sum:

$10 + 1 + 1/10 + 1/100 + 1/1000 \ldots = 11.1$ (recurring) $= 11\ 1/9$

which is the answer we would have got using an algebraic approach. We may similarly express one third as a decimal:

$1/3 = 3/10 + 3/100 + 3/1000 + \ldots = .3$ (recurring)

These are known as examples of convergent series. One of Cauchy's main contributions to mathematics was a set of criteria for determining if an infinite series is convergent or divergent.[18]

An example of an infinite series which is divergent is:

$1/2 + 1/3 + 1/4 + 1/5 + \ldots$

This series occurs if you build a tower of bricks (one on top of another), and try to build a curve into it. The top brick can be displaced half a brick from the one under it, it can be displaced one third of a brick from the one under it, and so on. We can prove that this series is not convergent as follows:

The smallest of the first 9 terms is 1/10, so they sum to at least 9/10,
The smallest of the next 90 terms is 1/100, so they sum to at least 9/10,
The smallest of the next 900 terms is 1/1000, so they sum to at least 9/10. . . . and so on, without limit.

However, until Cantor mathematicians were careful not to speak of actual infinite quantities. Galileo, Leibniz and Cauchy all rejected the use of the actual infinite.[19] Gauss wrote in 1831, "In mathematics infinite magnitude may never be used as something final; infinity is only a *facon de parler*, meaning a limit to which certain ratios may approach as closely as desired when others are permitted to increase indefinitely."[19] Descartes said, "The infinite is recognizable but not comprehensible."[19] One wonders if Descartes might not actually be paraphrasing the Bible. His statement seems to be in agreement with Ecclesiastes 3:11, which says, "He has set eternity in their heart, yet so that man will not find out the work which God has done from the beginning even to the end". The AV puts it a little differently, it reads "world" instead of "eternity", but the Hebrew word is OLAM which certainly suggests endlessness of space or time.

DENUMERABILITY

Not only did Cantor consider the actual infinite a valid mathematical entity, he defined methods of testing if infinite quantities are equal to one another, or one is greater than the other. He defined the smallest infinity as Aleph-null, with Aleph-1 a larger infinity, Aleph-2 larger again and so on.

His test for the equality of two infinities is known as one-to-one correspondence of the items in the two infinite sets, and gives rise to the term "denumerable". The reasoning runs as follows: Write down a list of the positive integers, and alongside each write its double, thus

1 2 3 4 5 6 . . .
2 4 6 8 10 12 . . .

It is seen that for every positive integer there is an even positive integer, because for any member of either set, it is a simple matter to find the corresponding member in the other set. Therefore, concluded Cantor, the two sets show one-to-one correspondence, or are denumerable, or equal. That is, there are as many even positive integers as there are positive integers (actually Aleph-null of them).

A somewhat similar proof may be constructed to show that the number of points on any line segment, is denumerable with the number of points on any other line segment (see Figure 1). c is presumed to equal Aleph-1. In Figure 1, the line CD is half the length of AB, but if we place them parallel, and construct the lines AC and BD to intersect at point O, it can be seen that if we choose any point on AB, and draw the line through this point and O, the resulting line intersects CD; defining a unique point. Thus for any point on AB there exists a corresponding point on CD, and vice versa. Thus the set of points on AB is denumerable with (or equal to) the set of points on CD.

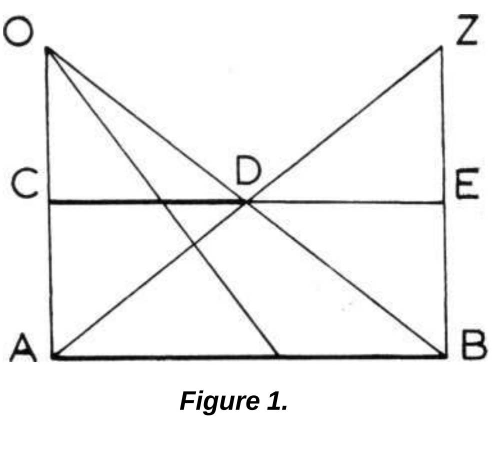

Figure 1.

However, I would question the validity of these proofs. In both the above cases the "proof is equally well a disproof. We could equally well construct the triangle ZAB, and proceed to show that there are as many points on DE as there are on AB, and since CE is the same length as AB they must each contain the same number of points, so CD does not contain the same number of points as AB, in fact CD contains infinitely less points (all the points on DE are missing). Similarly with the two infinite sets of positive integers: we know that in the second set, all the odd integers are missing, so how can we possibly conclude that the two sets are in one-to-one correspondence?

It is interesting that the mathematician Galileo considered a similar construction. Some authors claim that he anticipated Cantor's proof of equality, but I think the exact opposite is the case. Let's look at what he wrote. The work was **Discorsi e *dimonstrazione matematiche intorno a due* nuove** *sciense* written in 1638.[20] One character in this debate perceives the "denumerability" of the set of squares against the set of positive numbers. But his conclusion is: "I see no other decision that it may admit, but to say, that all Numbers are infinite; Squares are infinite; and that neither is the multitude of Squares less than all Numbers, nor this greater than that: and in conclusion, that the Attributes of Equality, Majority, and Minority have no place in Infinities, but only in terminate quantities ..." Galileo's conclusion seems to me to be far more reasonable than Cantor's idea that some infinities can be shown to be greater than others.

To believe our construction and reject the equally obvious opposite conclusion, is surely to follow the ancient Greeks, and depart from reality. Wouldn't it be safer just to conclude that an infinite set has the property of being able to be placed in one-to-one correspondence with a subset of itself? Therefore to accept infinite sets is to permit contradictions. Do we really need infinite sets anyway? The whole area seems to have contributed nothing to mathematics.

We will conclude this discussion on denumerability and orders of infinity, with a (typical humanist) quotation by Isaac Asimov — "So the human mind which painstakingly began by working out the difference between 1 and 2 has raised itself where it can fearlessly try to work out the difference between varieties of endlessness."[21] This despite the fact that he has admitted on the previous page that nobody has yet proved that aleph-1 equals the continuum c (see also Kline [22]). The human mind seems to have made remarkably little progress in this area since Cantor conceived the transfinite numbers in 1895.

BRIEF HISTORY OF MATHEMATICS

Let's now consider the traditional foundation of mathematics. All peoples from the time of creation would have had the God given sense of number, and an understanding of the basic arithmetic operations. Without this much, trade would not be possible. Adam would surely have had much more advanced mathematical knowledge than this, although we don't know just how advanced; but even where societies experienced cultural loss as a result of turning away from God, they must have still retained the knowledge of the basic axioms of arithmetic. By axioms we mean self-evident truths agreed upon by mathematicians. The basic axioms of arithmetic include the concepts of integer and addition, from which all the rest of mathematics can be deduced by logic. Such truths as 1 + 1 = 2, 2 + 1 = 3, 3 + 1 = 4 etc.

Our present array of mathematical techniques, however, seems to have blossomed at about the time of the Reformation. 'The seventeenth century is outstandingly conspicuous in the history of mathematics."[23] Logarithms were invented by Napier; Oughtred did significant development of algebra, and invented the slide rule; Descartes developed analytical geometry in about 1637; Newton and Leibniz independently developed calculus; Pascal did his foundational work in probability; and there was also the important work in mathematics related to astronomy by Galileo. Most of these men were Christians,[24] but most of the work done around that time, and continuing into the nineteenth century, would have been based on the rationale for science and mathematics which emanated from the rediscovery of the biblical basis. Nor do I suggest that there is anything wrong with any of these useful techniques.

In the latter half of the nineteenth century, however, with Darwinism becoming more acceptable, people started to question the number theory which was foundational to the whole structure of mathematics that had been built up; and sought to find a (humanistic) mathematical proof for the basic number concepts.

CANTOR'S SET THEORY

Cantor defined a set in 1895 as follows: "By a set (class) we understand any collection into a single whole of definite well-distinguished objects of our intuition or of our thought. The objects are called the elements (members) of the set."[25,26] Others have tried to find a better definition, but as yet nobody has stated any other definition that everybody is happy with.

The total number of sets that could exist by this definition must be infinite. Even if the universe consisted of nothing (except our thought), we could visualise a null set (containing nothing), then a set containing the null set, then a set containing this set, and so on. By this recursive construction we may build up an infinite number of sets.

Now what grounds would there be for inferring that experiences at a finite level could be consistently extrapolated to the infinite? Brouwer (who we shall look at later) suggested that some of the problems of mathematical reasoning might be deeply rooted in the uncritical extension to the infinite of a logic devised for the finite.[27] Poincare thought the theory of infinite sets a grave malady and pathologic. He said in 1908, "Later generations will regard set theory as a disease from which one has recovered."[28]

The main thing about set theory was that it was seen as a basis from which the concepts of number could be deduced. Frege, who followed Cantor's work, contributed much to the development of mathematical logic. He "proceeded in **Foundations of Mathematics** (1884) and his two-volume **Fundamental Laws of Mathematics** (1893, 1903) to derive the concepts of arithmetic and the definitions and laws of number from logical premises",[29] via set theory. It would seem that this is working backwards. God has given man understanding of the finite, and not the infinite; so to use the infinite as the starting point from which to prove the finite is clearly a risky business. Problems did in fact arise.

RUSSELL'S ANTINOMY

In 1902 Russell informed Frege of a contradiction ("antinomy" is a euphemism for "contradiction") which he had discovered in Frege's (and hence Cantor's) development. The second volume of Frege's **Fundamental Laws of Mathematics** contains this note at the end: "A scientist can hardly meet with anything more undesirable than to have the foundation give way just as the work is finished. In this position I was put by a letter from Mr Bertrand Russell as the work was nearly through the press."[30,31,32] What had gone wrong? At one point in the work, very early on, he had said: "Let S be the set of all those sets which are not members of themselves."[33]

Russell himself explained this paradox with the familiar barber paradox: the barber of a certain village has enunciated the principle that he shaves all those persons and only those persons of the village who do not shave themselves. The paradox arises when we ask the question "Who shaves the barber?" If he shaves himself, he falls into the category of those who shave themselves, and therefore he doesn't shave himself; but if he doesn't shave himself, according to his principle he has to shave himself.

Grelling's antinomy seems to be very similar: a few English adjectives, such as 'short', 'English' and 'polysyllabic', have the very same property that they denote, for example, the adjective 'polysyllabic' is polysyllabic, while the majority, such as 'French', 'monosyllabic', 'blue' and 'hot' do not. Calling the adjectives of the second kind **heterological,** we immediately discover to our dismay that the adjective **'heterological'** is heterological if and only if it is not heterological.[34]

From Cantor's definition of a set, it seems reasonable to think of the set of all sets, and the set of those sets which contain themself (of which the set of all sets is obviously one). So why should we not be entitled to think of the set of those sets which do not contain themself? However to do so, implies a contradiction.

So in fact the proof that 1 + 1 = 2, 2 + 1 = 3 etc. had fallen through. The proof that set theory was supposed to provide to place number theory on a solid foundation, had failed dismally.

FURTHER PARADOXES

After Russell's antinomy, several more paradoxes came to light.

The 'liar' paradox in its oldest known form, was stated by Eubulides in the fourth century BC. He said, "The statement I am now making is false."[35] Clearly his statement is true, if and only if it is false.

Consider all the positive numbers referred to (explicitly or implicitly) in this article. We can then refer to **the smallest positive number not referred to in this article.** But now the above bolded phrase refers to the smallest number not referred to. A contradiction!

Finally there is: Can an omnipotent being create a rock so heavy that he cannot lift it? If we reply either yes or no, we are denying our God's omnipotence.

We will answer at this point, only the last of these paradoxes. The God revealed in the Bible is not omnipotent in the full unrestricted sense. He cannot contradict His own nature; He cannot imply a contradiction, for example, "it is impossible for God to lie" (Hebrews 6:18), and "for He cannot deny Himself" (2 Timothy 2:13). Thus we also expect the creation to contain no contradictions. If we come across a contradiction, we see it as a danger signal that we have done something wrong, and we (that is, mathematicians with a biblical basis) must go back and find our mistake. I submit that the absence of contradiction provides the same function in mathematics as experimental verification provides in other sciences.

TWENTIETH CENTURY MATHEMATICS

The discovery of these paradoxes early this century, resulted in a re-examination of the handling of the set concept. This resulted in a great divergence in the diagnosis of the ills of Cantor's "Naive" set theory, and naturally the different diagnoses led to the recommendation of different cures.[36] Several philosophies were formulated:

Proponent	Philosophy/School
Russell /Whitehead	Logicist
Brouwer /Kronecker	Intuitionist
Hilbert	Formalist
Zermelo /Fraenkel	Set-Theoretic

These schools differ greatly in their contents. Not only that certain statements concerning sets are truths in one system and at the same time are falsehoods in another, but in many cases different systems use different languages, and there is not always a natural translation into the language of another system.[37] There was a definite humanistic tendency in all this, as evidenced by the fact that "Hilbert had unbounded confidence in the power of man's reason and understanding".[38]

Eric T. Bell wrote in 1930: "Knowledge in any sense of a reasonably common agreement on the fundamentals of mathematics seems to be non-existent . . . that equally competent experts have disagreed and do now disagree on the simplest aspects of any reasoning which makes the slightest claim, implicit or explicit, to universality, generality, or cogency."[39] "Thus by 1930, four separate, distinct, and more or less conflicting approaches to mathematics had been expounded, and the proponents of the several views were, it is no exaggeration to say, at war with each other. No longer could one say that a theorem of mathematics was correctly proven. By 1930 one has to add by whose standards it was deemed correct."[39] But worse was to come.

In 1931 Godel published what is known as "Godel's incompleteness theorem".[40] He showed, by methods acceptable to the followers of any of the philosophies of mathematics, that it is impossible for a sufficiently rich formalised deductive system, such as Hilbert's system, to prove consistency of the system by methods belonging to the system. He established the existence within the system of "undecidable" problems, of which consistency of the system is one.[41] This worried mathematicians, in that they could give no guarantees that further contradictions could not show up. One consequence of Godel's work was the realisation that faith was needed to establish mathematics — "Suppose we loosely define a **religion** as any discipline whose foundations rest on an element of faith, irrespective of any element of reason which may be present. Quantum mechanics for example would be a religion under this definition. But mathematics would hold the unique position of being the only branch of theology possessing rigorous demonstration of the fact that it should be so classified."[42]

Russell confessed in 1959: 'The splendid certainty which I had always hoped to find in mathematics was lost in a bewildering maze ... It is truly a complicated conceptual labyrinth."[43]

In 1946, Hermann Weyl, one of the outstanding scholars of the day said: "We are less certain than ever about the ultimate foundation of mathematics. Like everybody and everything in the world today, we have our 'crisis'. We have had it for nearly fifty years. Outwardly it does not seem to hamper our daily work, and yet I for one confess that it has had a considerable practical influence on my mathematical life: it directed my interests to fields I considered relatively 'safe', and has been a constant drain on my enthusiasm and determination with which I pursued my research work."[44,45]

Up until the present time, these fundamental questions have not been answered. The proof of number theory has not been forthcoming, and mathematicians in general have given up hope that it ever will. Of the different philosophies, none has been triumphant over the others, but can we say that any one is nearer to the truth than the others? I think we can.

THE INTUITIONIST SCHOOL

It seems to me that the Intuitionist philosophy of mathematics comes closest to the basis we should expect. "Kronecker was the first of a distinguished group who felt that one could not build up more soundly by logical means what man's intuition assured him was sound".[46] Although Intuitionism can be traced back at least to Descartes and Pascal,[47] this line of reasoning was brought to the fore mainly by Brouwer; in his doctoral dissertation "On the Foundations of Mathematics" (1907) he began to propound the intuitionist philosophy.[48] It takes the axioms of arithmetic as proved by intuition: 'The nucleus of positive intuitionistic principles, common to all intuitionistic trends from Kronecker to the present day, is the 'primordial intuition' of **positive integer** or of the construction by **mathematical induction".**[49] It rejects the suggestion that mathematical induction might be considered an ingredient of a **definition** of integers.[50] In other words, logic does not predate number concepts. Kronecker said, "God made the integers, everything else is the work of man."[50,51,52] Indeed Hilbert's criticism of the intuitionist philosophy was: "To found it [mathematics] I do not need God, as does Kronecker, . . ,"[53]

"Brouwer's chief doctrine was the denial of the law of the excluded middle".[54] The law of the excluded middle is usually taken to mean that every proposition is (and is provable as) either true or false. It is clear that this restriction removes the problem of the 'liar' paradox. Further "Kronecker insisted that unless we can give a definite means of constructing the mathematical things about which we talk and think we are reasoning, we are talking nonsense and not reasoning at all".[55] "The fundamental thesis of intuitionism in almost all its variants says that existence in mathematics coincides with constructibility".[56] "Brouwer, following Kronecker, demanded constructions for mathematical 'entities' whose 'existence' is purportedly proved without giving any method for exhibiting the 'entities' in a **finite** number of humanly performable operations."[57] This clearly solves some of the contradictions, but it seems to me to be too extreme: in 1882 when it was proved that π is transcendental, Kronecker said "Of what value is your beautiful proof, since irrational numbers **do not** exist?"[58,59] Perhaps we should allow some intermediate criterion for a proof of existence.

This philosophy further believes that "Traditional mathematics has misinterpreted and mismanaged the **concept of infinity",**[60] and that the contradictions in set theory are due to the abundant and unlimited use of infinity.[61] Godel's incompleteness theorem is recognised by some authors as an argument in favour of intuitionism (e.g. Kline [62]) because it shows that some propositions are neither provable nor disprovable. Finally we note that unlike other philosophies, intuitionism does not try to construct the continuum. The continuum is presumed to exist from the first as a basis, although there are various opinions on how it should be accepted.[63]

COMPUTERS

This criticism of modern mathematics does not reflect on computers. Whatever reservations we might have about computers, they do not implement twentieth century mathematics. Twentieth century technology they may be, but they definitely implement pre-Cantorian mathematics.

Computers cannot handle infinity: they allow only a limited range for integers, and division by zero causes an error condition. Real numbers are usually approximated. Most computers use the binary system so that any powers of 2 will be represented exactly (e.g. 1/8), but other real numbers will be approximated, whether rational or irrational.

Computer programming is a creative activity. If it is required to add up 1/10 ten times, the programmer has to realise that the answer will not be exactly one. Certain calculations can be done only if the programmer is careful, for example, sin(X)/X when X is zero. The result should be obtained as close to 1 when X is close to 0.

CONCLUSIONS

No satisfactory proof has ever been devised to show that the basic number theory is universally true; rather proofs have been put forward indicating that such a proof may be impossible. Thus, mathematics cannot contradict the scriptural teaching that God exists as three persons in unity. However, the number theory can and should be accepted (it is according to the intuitionist approach) as being correct for this creation. This was my main motivation in researching this material, but a number of further conclusions have become clear to me in the process.

Modern mathematics is in a mess, searching for its foundations. While this modern crisis has so far been largely hidden from the general public, we should not assume it will stay that way. Mathematics needs to return to its God-given basis as set down in the Holy Bible. To any readers involved in mathematics, I would urge that you make a serious study of the biblical basis for mathematics. It would seem to me that the intuitionist philosophy comes closest to providing a firm foundation.

Ever since the thirteenth century (when Thomas Aquinas taught that all learning could be divided into religious knowledge and natural knowledge), the Church has been willing to leave mathematics to the ''experts". In doing so we have forgotten the revelationist basis of mathematics. That mathematics has a religious basis is even admitted by present day mathematicians. And what is mathematics anyway? Perhaps we should define it thus: mathematics is man's attempt to understand the patterns in God's creation, and to exploit those patterns in our God-given function of administering the creation.

If we reject the use of the "actual" infinite in mathematics, we are in company with some of the great mathematicians of the past. What's more, we are following the guidance given by scripture in such passages as Ecclesiastes 3:11 and Psalm 115:16.

"In pure mathematics we contemplate absolute truths, which existed in the Divine Mind before the morning stars sang together, and which will continue to exist there, when the last of their radiant host shall have fallen from heaven." So spoke E. Everett in November 1863.[64] When this was people's perception, mathematics was on a firm foundation. If mathematics is based on God's revelation we can be certain about the consistency and correctness of our mathematics, because we can be sure about our God. I suggest that certainty exists on no other basis.

MATHEMATICIANS MENTIONED

<u>Brouwer</u>, Luitzen E.J. (1881-1966). Dutch professor of mathematics. The principal advocate for the modern Intuitionist philosophy.

<u>Cantor,</u> Georg Ferdinand Ludwig Philip (1845-1918). Born in Russia of Danish parents. Spent a long teaching career at the University of Halle, Germany, from 1869 to 1905. Died in a mental hospital.

Cauchy, Augustin Louis (1789-1857). Developed techniques for determining the convergence of infinite series.

<u>Descartes</u>, Rene (1596-1650). Responsible for analytical geometry.

<u>Euclid</u> (circa 300 BC). Professor of mathematics at Alexandria. Collected the totality of Greek geometry (etc) into the "Elements".

<u>Fraenkel</u>, Abraham A. (1891-1966). Professor of Mathematics, Hebrew University of Jerusalem.

<u>Frege</u>, Gottlob (1848-1925). German logician. He worked to establish what he considered a sounder foundation for mathematics.

<u>Fourier</u>, Joseph (1768-1830). Held a teaching post at the Ecole Polytechnique in Paris. Wrote material on the mathematical theory of heat conduction. Invented Fourier series etc.

<u>Galileo</u> Galilei (1564-1642). Appointed professor of mathematics at Pisa at the age of 25. Well known for his astronomical discoveries.

<u>Gauss</u>, Carl Freidrich (1777-1855). Regarded in his lifetime as Germany's greatest mathematician. Contributed to many branches of physics.

<u>Godel</u>, Kurt (1906-). Austrian logician.

Hilbert, David (1862-1943). Attempted a formal axiomatization of mathematics.

<u>Kronecker</u>, Leopold (1823-1891). Taught at the University of Berlin, and was one of Cantor's most outspoken opponents.

<u>Leibniz,</u> Gottfried Wilhelm (1646-1676). He developed calculus between 1673 and 1676. He first used the modern integral sign.

Napier, John (1550-1617). Invented logarithms. Supporter of John Knox.
Newton, Isaac (1642-1727). Studied at Cambridge University. Devised differentiation and solving of differential equations. His greatest work was "Principia".
Oughtred, Wiliam (1574-1660). Episcopal minister. Invented the slide rule.
Pascal, Blaise (1623-1662). In about 1642 he invented the first computing machine. He also did a lot of work on probability, and conies.
Poincare, Jules Henri (1854-1912). Held several professorships in mathematics and science at the University of Paris from 1881.
Riemann, Georg F.B. (1826-1866). Responsible for Riemannian geometry.
Russell, Bertrand (1872-1970). Philosopher. He followed on from Frege, and co-authored the influential work 'Trincipia mathematica".
Whitehead, Alfred North (1861-1947). Co-authored 'Trincipia mathematica".
Zeno (circa 450 BC). A Greek philosopher. He propounded several paradoxes which seem to show that motion is impossible.
Zermelo, Ernst (1871-1953). Propounded the set-theoretic approach.

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I have made most use of Kline, Bell, Fraenkel et al., and Eves (not that they agree with my point of view). The deepest and most comprehensive of these is Fraenkel et al.; but it would be almost unintelligible to me if I had not read one of the others first.

For anybody wanting to pursue set theory, Fraenkel et al. seems to give a very good summary of the current situation, and lists a 40 page bibliography.

FOOTNOTES

Answers to Objections, and Suggestions for Further Work

1. INTUITIONISM. I agree that steps need to be taken to cure the antinomies But the measures taken by the other philosophies seem to me to be arbitrary and somewhat ad hoc. Any solution that gave the intuitively (!) obvious answer was seized upon. Only the solution proposed by intuitionism (which actually predated the problem) seems to be based on the Bible. I don't think the Intuitionist philosophy is totally correct. I would say there was a pervading humanist influence in all the different philosophies, but Intuitionism alone has some sort of revelationist basis.
2. AXIOMS. Does 1 + 1 = 2, 2 + 1 = 3 etc.? I believe these axioms are proved by the fact that they are intuitively obvious to man, as applied to this creation; but unprovable elsewhere (e.g. other possible creations?). God is revealed in the scriptures (OT and NT) as Three Persons in one Godhead, which is quite fully set down in the Athanasian Creed. This is something we cannot understand. It seems to be a good test of which are Christian denominations and which are sects. Christian teaching concludes that here is a profound truth we don't understand, while the sects exalt human reason above revelation and reject the teaching that God exists as Three in One.
3. INFINITY. It is necessary to have an infinity such as defined by Gauss (Kline, p.200) to avoid contradictions (like Zeno's paradoxes), but Cantor's "actual" infinity engenders contradictions, and has produced nothing useful. We should bear in mind that when we talk about infinity we are stretching the bounds of human comprehension.
4. INFINITIES. The standard argument for orders of infinity says:

   i) There are more irrationals than rationals. Proof: π is irrational, therefore is not included in the set of rational numbers. Therefore the order of the infinite set of irrationals exceeds the order of the infinite set of rationals; because we have shown there is at least one item which has no corresponding item in the other set.

   Note that we could have used exactly the same reasoning to show that there are more rational numbers than there are integers:

   ii) The rational number 1/2 is not included in the set of integers; therefore the order of the infinite set of rational numbers exceeds the order of the infinite set of integers.

   Proofs (i) and (ii) are exactly the same. However, it is known that the second proof is fallacious; because constructions have been devised to place the rationals in one-to-one correspondence with the integers. So why do most mathematicians accept the first "proof"? The difficulty is:

a) An infinite set can be placed in one-to-one correspondence with a proper subset of itself (Cantor).
b) The use of infinite sets will lead to contradictions (Bolzano).
c) $< \; = \; >$ have no meaning with respect to infinite sets (Galileo).

Therefore any argument attempting to prove the non-denumerability of one infinite set against another infinite set (e.g. the continuum of real numbers against the set of positive integers) is meaningless.