

# Does logic need faith?

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God created abstract things in addition to the material. This includes logic and all its inferences. In 1931 Kurt Gödel proved a seminal theorem in formal logic that has far reaching ramifications in all systems based on axioms or assumptions. One of the most important conclusions of Gödel's proof is that faith (a non-self-evident axiom) is a necessary precondition if any such system is to maintain consistency, and if it is consistent, it must necessarily be incomplete. This has far reaching results in philosophy and matches the patterns of meaning in many scriptures.

‘Through him [the Word, God] all things were made; without him nothing was made that has been made.’ (John 1:2 (NIV)).

## Scientism and post-modernism

The majority of people reading *all things* think of concrete, material objects: things such as rocks, water, the sun and moon. But this is because our reasoning has been largely colored, indeed saturated, with the philosophy of *materialism* and its modes of thought, which have been gaining momentum since the mid-nineteenth century. But *all things* must also include such abstractions as reason itself: have you ever stopped to consider what *to consider* really is? But abstract things are now deemed to be unreal. Never mind trying to mention *spiritual things* in a polite conversation with a modern secular scientist. He will simply dismiss your arguments in favour of creation as ‘religious’ or at best ‘subjective’ (he really means *unreal*) as opposed to his being ‘objective’ (read *real*). Does he have a point? After all, science claims to be dealing with the material world exclusively. Science, he will tell you, has nothing to do with faith and everything to do with material evidence and its objective, logical analysis. Is he right? It depends on to what extent he believes that *Science* is the answer to all questions. I am distinguishing here between science (little ‘s’) as a methodology and *Science* as a philosophy. *Science*, when practiced as a philosophy, goes incognito under names such as *Rationalism*, *Positivism*, *Pragmatism* and others. But boiled down, these worldviews always mean a belief that the material world is all there is, and material itself contains all answers. All truth may be found if we just look hard enough or receive enough grant money. Barzun calls this ‘Science as worldview’ Scientism, which

‘is the old Huxleyan belief that nothing is outside the scope of science and that it can furnish answers to all human problems. It goes hand in hand with the Marxian denial of the efficacy of ideas, and it makes science a substitute for philosophy, art and religion.’<sup>1</sup>

Post-modern thinking derives in large part from three leading exponents of materialism who became icons after 1859: Charles Darwin, Karl Marx and Richard Wagner. Although all three promoted themselves as pursuing original ‘scientific’ methods, none did much in the way of real science and all three did very much in the way of philoso-

phizing. Ironically none of these men were truly original, but combined and adapted ideas developed earlier, mostly from the Enlightenment period of the latter half of the 18<sup>th</sup> century. Darwin’s cleverly chosen word *Origin*, in his book’s *Origin of the Species* title, ultimately implied that life arose—evolved—from the process of natural selection, but he never came close to proving any such thing. Instead, *Darwinism*, as his philosophy became known, became a necessary presupposition to enable Nazism, racism, euthanasia and other abominations to flourish.<sup>2</sup> Marx’s terrible and incorrect theories led to the horrors of communism, and yet the so-called science he used to support his philosophy was entirely bogus. Wagner became the messiah of art as worship. Of course, being the artist, Wagner expected and received this worship. The supplanting of truth with artistic mythology by the culture of Hollywood is only part of Wagner’s legacy. Bayreuth was Wagner’s ‘tinseltown’. The corrosive ramifications of the materialist philosophy practiced by these three men are manifest today.

## Science and philosophy—arenas for Christian witnessing?

Since *Science* and *Philosophy* are now so hopelessly polluted with secular beliefs, are they really the proper arenas for Christian witnessing? Not while people who believe in *Science* (materialism) continue to believe that they alone are objective; they alone grasp true reality and don’t need faith’s ‘crutch’ to do their work. Naturally, we tend to think that to have any hope of *persuading* someone to ask Jesus to be his Saviour, we have to find some common ground. I am reminded of Paul at the Areopagus in Athens (Acts 17:16ff.). He spent a great deal of effort trying to reason with skeptics, but in the end only ‘a few men became followers of Paul and believed’ (Acts 17:34). Hardly a prolific result, and anyone who has attempted to open a dialogue about creation with a hardened secular scientist ensconced in evolutionary thinking will know why. There seems to be much more hope for successful reasoning with people who are already skeptical about evolution, as a result of realizing that there is no comfort in it, no joy, and no future. I believe it is because of their desire to escape these fearful pressures, and not so much from scientific reasoning, that ‘creation evangelism’ works. Nevertheless, the hard cases must not be ignored. How do we find a common ground?

Secular, materialistic skeptics present themselves as having superior reasoning powers than we Christian creationists do because they claim not to need *faith*. *But what if it could be proved that the logic so necessary to scientific methods could not proceed consistently without faith?* If that were true, then everything would reduce to ‘religion’ and we might have a common ground of sorts. Creationists often state that evolution is a (secular) religion. Evolutionists will of course disagree. The creationists are right, but how to prove it? First, we will have to seek a common ground that has no value judgments attached, no preconceived meanings, or else our discussion will degenerate into shouting slogans at worst or a meaningless discussion of linguistics at best.

### Formal Logic

Something we call *Formal Logic*, it turns out, is the only common ground between the creationist and the evolutionist *if we limit ourselves to scientific methods only*. And there happens to be a theorem and a logical framework we can use to prove our point.

The seminal proof in Formal Logic that we need was published at Vienna, Austria in January 1931. The author was a 24-year-old German-speaking mathematician named Kurt Gödel. The paper, little understood or appreciated at the time, was called ‘On formally undecidable propositions of *Principia Mathematica* and related systems I’. Gödel has been called the greatest logician of the 20<sup>th</sup> century, but fully appreciating his genius and his work falls to only a handful of mathematicians. While understanding his work as presented is difficult because of its abstruse mathematics, it is possible for the non-mathematician to understand something of the ramifications of *Gödel’s Incompleteness Theorem* as it became known (we will refer to it simply as Gödel’s proof from now on). My intention here is not to explain the proof, but rather to present some of its ramifications. I will try to present a simplified explanation of what Gödel proved, enough to allow us to explore some interesting terrain together.

Formal logic has to do with *axiomatic systems*. If you recall your high school geometry, an axiom is, loosely speaking, a ‘self evident’ truth. We can also take an axiom to mean an agreed upon precondition or assumption. From axioms we prove *theorems*, which are the logical consequences from analyzing the underlying axioms. If we follow the rules of logic correctly, a *proof* consists of being able to *derive* a theorem from one or more axioms (or previously proven theorems) by following a finite number of logically true steps. An *axiomatic system* is composed of a *set* of axioms from which we derive the theorems of the system. Euclidean geometry is a good example of an axiomatic system.

Most scientific reasoning depends on the logical progression from initial conditions or assumptions. Scientific methods usually follow some mathematical model, which is itself an axiomatic system. Many scientific breakthroughs happen because the mathematical model predicts some result later confirmed by experiment. So we may think of formal logic as a framework on which to place meanings and values, but there is no need to place any values *a priori* in order to

discuss Gödel’s proof. To help us visualize things better, I want to characterize and paraphrase in geometry (most of us have had a course in high school geometry) what Gödel did using number theory.

In Euclidean geometry, one of its foundational axioms is

1. ‘We have a line and a point outside the line; there can be *only one line* parallel to the given line through that point.’  
We recall that an axiom is ‘self evidently’ true, but is the parallel axiom always true? What if we vary it to exhaust all the possibilities? Thus
2. ‘We have a line and a point outside the line: there are *no lines* parallel to the given line through that point.’
3. ‘We have a line and a point outside the line: there are *infinitely many lines* parallel to the given line through that point.’

We all know that Euclidean geometry is useful and we can build bridges and skyscrapers using it. We also know Euclidean geometry to be consistent. It contains no contradictions. But amazingly, the other two (2 and 3 above) lead to equally consistent geometry. Number 2 is an axiom of what we call Riemann or elliptical geometry and number 3 is an axiom of Hilbert spaces. We can conclude that all three of these geometric (axiomatic) systems contain one axiom that is *not* self evident which contradicts the very definition of *axiom!*

### Formal system

For our purposes it is convenient to paraphrase Gödel’s proof by saying he generalized the finding above: every consistent axiomatic formalized system must by necessity have a foundational axiom that is *not* self-evident! This statement may prove easiest for readers to grasp. I am interested primarily in the ramifications rather than in the methods of proof, but it is important to understand at least the gist of what Gödel reveals about how we use logic and reasoning and their limits. After digesting several definitions, we can have a look at a brief summary of what Gödel proved.

When a system is completely formalized, it is completely drained of all meaning; its axioms and theorems are just strings of meaningless symbols connected in such a way that they obey a set of transformation rules as to what a valid theorem is. When we say a system is *consistent*, it means that *both* a proposition (or theorem, or formula) P, and the negation of P (we write that  $\sim P$ ) cannot be derived from the axioms. In other words there are no contradictions. We need to make a distinction between mathematics (deriving theorems for instance) and when we are discussing mathematics. We could say, ‘Mathematics is *mathematics*, what we *say about* mathematics is *metamathematics*’. The mathematician David Hilbert, a contemporary of Gödel, coined the term. *Demonstrable* means that a proposition can be derived from the foundation axioms in a finite number of steps using the transformation rules and logical rules of inference. An isomorphism is a mapping between two domains, call them A and B. If A is isomorphic to B, then the map implements a

one-to-one correspondence without distortion. From A, you can go to B, do things, and get back to the equivalent point in A as if you had done the equivalent things in A. *Principia Mathematica* is a three-volume work by Alfred North Whitehead and Bertrand Russell on formal logic and the foundations of mathematics. Among other things it introduced the concept and notation of the completely formalized axiomatic system of numbers. We will call this system PM, but it can stand in for any axiomatic system. Finally, we have the concept of *self-reference* (in mathematics it's called *recursion*). The Liar's Paradox is a famous recursion: 'This statement is false'. It says something about itself: it declares its own falsehood, paradoxically. If we take the statement as true, it declares itself false, but if we take it to be false then it is true. A paradox is *undecidable*. As we will see shortly, recursion and paradox are at the heart of Gödel's reasoning.

### Gödel's Incompleteness Theorem

Gödel's proof<sup>3</sup> starts with PM. PM, being completely formalized allows one to characterize all relationships, theorems and proofs as strings of symbols put together according to transformation rules. He found an isomorphism between PM and the cardinal numbers (these are the counting numbers or set of positive integers) that allowed him to map symbols, theorems (or formulas) and proofs (a proof is a series of theorems) in PM to (very large) unique integers. Conversely his mapping can extract the PM propositions, variables, symbols, theorems, proofs and so forth, intact from these large integers. These large integers are called Gödel numbers. And now for the brilliant part: Gödel discovered a way to characterize metamathematical statements as formulas of PM



My 1966 painting 'Portrait of the artist as Kurt Gödel' demonstrates the humour of self-reference, as well as the tension between the concrete, the abstract, and the logical.

and map these to Gödel numbers as well. The system now has the capacity of *self-reference*; statements *about* PM can be characterized *within* the system itself. At this point Gödel introduced something akin to the Liar's Paradox ('This statement is false'). He created a metamathematical statement G with Gödel number g. He constructed G so that it says 'The formula that has Gödel number g is not demonstrable' (which means derivable using the rules of PM). But

then he showed that G is demonstrable if and only if  $\sim G$  (its negation) is demonstrable. This means that PM is not consistent. Conversely, if PM is consistent, then neither G nor  $\sim G$  can be formally derived from the axioms. This is what is meant by *G is formally undecidable*. But Gödel shows that G (as characterized in PM) is a *true* arithmetical statement even though it is *undecidable* within PM. Therefore PM is *incomplete*; that is, there are true statements that cannot be derived from the axioms of PM. Gödel further proved that PM is *essentially incomplete*, which means that adding to the set of axioms will never make PM complete. Finally, Gödel demonstrated how to construct a formula A of PM that characterizes the metamathematical statement 'PM is consistent'. Then he constructed the formula 'A G'. This formula is composed of PM symbols which mean 'if A then G'. Substituting, we get 'If PM is complete, then the formula that has Gödel number g is not demonstrable'. Gödel proved that formula 'A G' is formally demonstrable inside PM, but A is *not* demonstrable inside PM. Therefore, the consistency of PM can never be deduced using the rules of PM and its axioms.

'Now faith is being sure of what we hope for and certain of what we do not see' (Hebrews 11:1 (NIV)).

'And without faith it is impossible to please God, because anyone who comes to him must believe that he exists and that he rewards those who earnestly seek him' (Hebrews 11:6 (NIV)).

'"Come now, let us reason together," says the Lord' (Isaiah 1:18a (NIV)).

### Faith is necessary

We now come to my main conjecture: Gödel's Incompleteness Theorem *proves the necessity for the existence of faith*. Gödel's concept of *undecidable within the formal system* has a profound sense of 'can't get there from here using only logical reasoning and staying within a given system'. The Scriptures lead me to make the conjecture that Gödel's result is generally true in a wider domain. While science is rarely done using formal systems of meaningless strings of symbols, it is true that all scientific methods begin with assumptions as in an axiomatic system. Scientism parades itself as the secular god of all knowledge, but in fact every one of its disciplines reduces to a very limited system based on what are essentially axioms plus experimentation. That much knowledge comes of this is undeniable. But the problem is a philosophical one: the belief that eventually all knowledge can be had by *staying within* the system is supremely arrogant and untrue. Secular scientists roll their eyes at the word *metaphysics* (because it smacks of 'religion'), but just as with *metamathematics* in Gödel's proof it is essential to the consistency of the system that the 'outside' truth exists.

It is interesting that Gödel was brought up a Lutheran and spent a great deal of time contemplating theology. He believed in mathematical *Platonism*.<sup>4</sup> This is the belief that mathematical objects really exist as ideal archetypes and are

there to be discovered by our intellect. Later in life, in some notes on the philosophical importance of his Incompleteness Theorem, Gödel expressed opposition to Alan Turing's mechanistic view of mind.<sup>5</sup> Secular scientists always choose the material and the mechanistic because they believe that staying within the system is the very definition of science, and maybe it is. But they are ignoring the rich and fruitful realms of inspiration, intuition and divine revelation. Those with a more spiritual frame of reference believe that God made the material as well as the abstract and spiritual and mathematical things—He made all things. And these things are waiting for us to discover them. I think Gödel was correct. In Scripture we have:

‘It is the glory of God to conceal a thing; but the honour of kings to search out a matter’ (Proverbs 25:2 (KJV)).

### Patterns in Scripture

Let's assume that Gödel was right: that he *discovered* something—this *Incompleteness*—that was there all along.<sup>6</sup> It would be interesting to see if what we know of Scripture, religions, the history of ideas, and our own experiences would fit well with this *Incompleteness* and suggest that Gödel's findings are true *in general*.

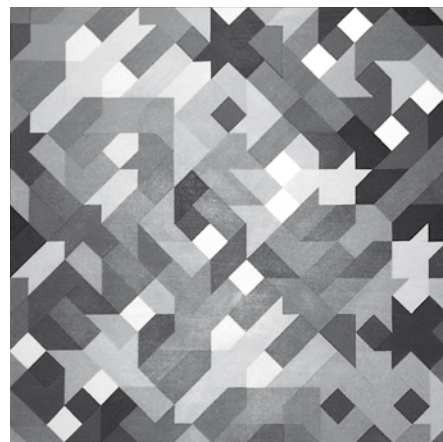
John 1:1 says that the *Word* is God. Interestingly the Greek for *Word* is *Logos*, from which root we also get our word *logic*. I conjectured above that *faith* is equivalent to *accepting a true statement that is undecidable within the formal system*. Now since we Christians believe that God created all things, He therefore created *Logic* and all its powers of inference. So, if we mix things up a bit, substituting, we get a true statement: ‘Without faith, it is impossible to please (that is, satisfy) Logic.’ While this statement exists in an infinitely smaller realm than Hebrews 11:6a, the logical patterns of the two statements certainly match (IF  $\sim A$  THEN  $\sim B$ ). The statement does not convey the meaning the original author of the Bible verse intended, and we cannot claim it to be an exegesis. Nevertheless, we do find intriguing similarities between Gödel's formula G (a true statement, but undecidable) on PM (*Principia Mathematica*, or any formal system) on the one hand, and God (*G*) in relation to PM (Postmodern Man's philosophy of scientism), on the other. Gödel's construction somehow fits the spiritual reality that we take on faith and that the world does not. We know God as the Eternal Other: He is self-existent, dwells in unapproachable light, is The Truth, His ways are not our ways, etc. Yet he says, ‘Come let us reason together’. He condescends to set us free when we accept His Truth in faith. Isn't freedom in fact being free of contradictions and inconsistencies? Accepting His Truth into our ‘system’ makes us *consistent*. But we (and our systems) are always *incomplete* without Him. No matter how much we strive, we cannot know or reach Him on our own, within the world system. Conversely, every worldly philosophy and false religion seeks completeness, and by so doing becomes inconsistent. Evolutionism, Nazism, Marxism, and Secular Humanism (to name only a few ‘ism's’), seek to have a

‘complete’ system, but are riddled with inconsistencies. It always goes against the sin nature of prideful man to admit his need for Truth outside what he can assume, invent, and deduce on his own.

### Experiences with Gödel's proof

Finally, we have to ask, ‘Will any of this work? Will this long argument persuade anyone that they need to exercise faith toward God?’ I know of only one instance: me. I can truthfully say that reading *A Primer of Gödel's Proof* led me on the first steps to becoming a real Christian believer. This small book is no longer in print<sup>7</sup> and was lent to me by a friend, but it changed my life. My friend and I had no notion that it was anything but a mathematics book about an interesting and curious subject. My problem was that I had been educated to believe that faith was some kind of ‘hocus pocus’ that has no place in a rational mind. In other words, I was steeped in the secular worldview of Scientism. Looking back, I see that Gödel's proof gave me a sort of ‘permission’ to have faith by proving that I *must* have faith. I saw not only that faith is unquestionably an inseparable part of formal deduction, but I also made the intuitive leap to believe that it is generally true in all systems including philosophical ones. If I am right in generalizing, the remarkable thing is that by Gödel's proof I cannot *prove* that I am right (within human reasoning). But at the same time the most rabid skeptic cannot *disprove* my stance. God in His wisdom made it this way. I am only saved through faith, and this not of myself, as faith (and the Truth it brings to bear) is the gift of the Lord. This is the way it must be. Scripture is not God. The Bible is only consistent if we allow the Holy Spirit to lead us into all truth. Systematic Theology cannot save you. God (the only Saviour) always transcends any system, even a theological one. Faith in revealed truth, through a relationship rather than information and analysis, is the way we must go.

I mentioned one other thing earlier, about every system reducing to a ‘religion’ because of Gödel's results. Unfortunately, this agrees with my experiences with practitioners of Scientism in person and in print. Again, by Gödel's proof it is impossible to reason a way to a conclusion common to everyone—we must choose. I have had no success in using this argument with a couple of secular



My 1987 painting ‘Encoded Symmetry Group with Imperfection’. The mathematical entity (a symmetry group) is not just represented by the painting; the painting is in reality the mathematical entity. This kind of self-reference is related to Gödel's Proof.

physicists whom I know personally. It did amuse me at one point, however, as I was speaking only in terms of formal logic and metamathematics. I must have backed my friend into a ‘logical corner’ because he asked me to ‘stop preaching’. This encourages me that there must be some spiritual ramifications worth exploring in Gödel. Mostly, I wanted to address other Christians who may be interested in the relationship between logic and faith in their own walk.

### Conclusion

I find it sad, but predictable, that those who have taken great pains to write books explaining Gödel’s proof should reach conclusions so different from mine. Invariably they espouse the mechanistic, materialist views that Barzun exposed in his excellent book. Even Barzun, after incisively revealing the problem with materialism and rationalism and the outcomes for those who practice them, falls into the same trap. In an attempt to be complete, to offer a well reasoned answer, he resorts to human wisdom alone, and his answer ends up just as inconsistent and incoherent as the doctrines of the three men he criticizes. As long as humanity reasons in a vacuum, no matter how skilled its thinkers or how sophisticated the reasoning, human effort will fail.

‘All the words of my mouth are just; none of them is crooked or perverse. To the discerning all of them are right; they are faultless to those who have knowledge’ (Proverbs 8:8–9 (NIV)).

‘Good understanding wins favour, but the way of the unfaithful is hard’ (Proverbs 13:15 (NIV)).

‘... Who then, can be saved? Jesus looked at them and said, ‘With man this is impossible, but with God all things are possible’ (Matthew 19:25b, 26).

How could anyone *not* want to accept the wisdom, insight and truth our Lord Jesus offers us? It wouldn’t be logical, now, would it? But it would have to be taken on faith ...

### References

1. Barzun, J., *Darwin, Marx, Wagner: Critique of a Heritage, 2nd ed.*, Doubleday, Garden City, NY, p. 335, 1958.
  2. cf. Weikart, R., *From Darwin to Hitler*, Palgrave Macmillan, New York, 2004, for details of the influence Darwinism had on the Nazis; how the ideas in Darwinism led to their devaluation of human life. He indicates that a Darwinist worldview was a *necessary but not sufficient* condition for what the Nazis did, meaning not every Darwinist will inevitably become a ‘Hitler’, but every ‘Hitler’ will have a Darwinist worldview. *Barzun*, originally writing prior to 1941, did not possess the facts regarding either the Nazi holocaust or the Stalinist purges. What he did discuss is the common theme of *continuous struggle* in the philosophies developed by Darwin, Marx and Wagner. The fact that Hitler’s struggle was against ‘racial impurity’ and Stalin’s purges were against ‘political impurity’ does not change the nature of the game. Marxism devalues human life as much as Darwinism.
  3. Nagel, E. and Newman, J.R., *Gödel’s Proof, revised ed.*; edited and with a new foreword by Hofstadter, D.R., New York University Press, pp. 92ff, 2001. This is the best available book for the non-specialist to use in tracing the highlights of Gödel’s Incompleteness Theorem.
  4. Feferman, S., (Ed.), *Kurt Gödel Collected works, Vol. 1: Publications 1929–1936*, Oxford University Press, Inc., NYC, p. 25, 1986. This contains the actual text of Gödel’s work in German, with translations in English, a biography and extensive notes. While the mathematics is not comprehensible to the non-specialist, the extensive notes on the theorems and the biography of Gödel provide valuable insights into his remarkable genius.
- NB: The word Platonism may be somewhat unfortunate. Rather than some other debatable ‘ideal’ as defined by Plato, here Gödel is referring to an inseparable component of cognition: logic itself, which is different in kind from other ‘ideals’. To those who say we must always interpret scripture solely with scripture, then Proverbs 25:2 should suffice (see main text). Those whose Christian apologetics derive from Justin Martyr (b. AD 165) will not have a problem, as he accepted whatever Greek thought meshed with Christian beliefs. (cf. González, J.L., *The Story of Christianity, Vol. I and II*, Prince Press, Peabody, MA, pp. 54 ff, 2004). But to those whose apologetics hark back to Tertullian of Carthage (AD 195–220), then let him speak for himself. While Tertullian is famously quoted:
- ‘*Quid ergo Athenis et Hierosolymis.* (What has Athens to do with Jerusalem?)’
- NB: Context is that importing secular ideas into Christ’s teaching is mixing chalk and cheese together.
- He also said:
- ‘*Quippe res dei ratio quia deus omnium conditor nihil non ratione providit disposuit ordinavit, nihil [enim] non ratione tractari intellegique voluit.* (For reason is a property of God’s, since there is nothing which God, the creator of all things, has not foreseen, arranged and determined by reason; moreover, there is nothing He does not wish to be investigated and understood by reason.)’
- There are 340 passages in Tertullian where the word *ratio* appears, making it one of the most frequently used nouns in his work; <[www.tertullian.org/quotes.htm](http://www.tertullian.org/quotes.htm)> items 6 and 3 respectively, 26 June 2005.
5. Feferman, ref. 4, p 13.
  6. This is not so much of a stretch as some may say. Gödel proved beyond a doubt that if a system can be mapped isomorphically onto *arithmetic*, it *must* obey his incompleteness theorem. Now believers in Scientism make the claim that this mapping to arithmetic is somehow too limited, but they have an axe to grind. In their rejection of God, they strive to prove the existence of such things as artificial intelligence, which they claim, can evolve along with computer science. But ultimately all of computer science reduces to arithmetic (binary at that!) no matter how elaborate the algorithms become. If we have learned nothing else from computer science, it is that arithmetic is generally true. No one takes seriously anything that maps onto ‘two plus two equals five’. To the champions of artificial intelligence, I say, ‘Only human beings are capable of artificial intelligence’.
  7. *Nagel and Newman* is a good treatment, but it does not contain the more readily visualized geometric explanations I originally found in *A Primer of Gödel’s Proof* (no longer in print). I remember vividly the variations of the parallel axiom of geometry from this little book. Yale University Press published it sometime in the 1960s. My explanation using the ‘non self-evident axiom’ in this paper is recalled from that work.

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