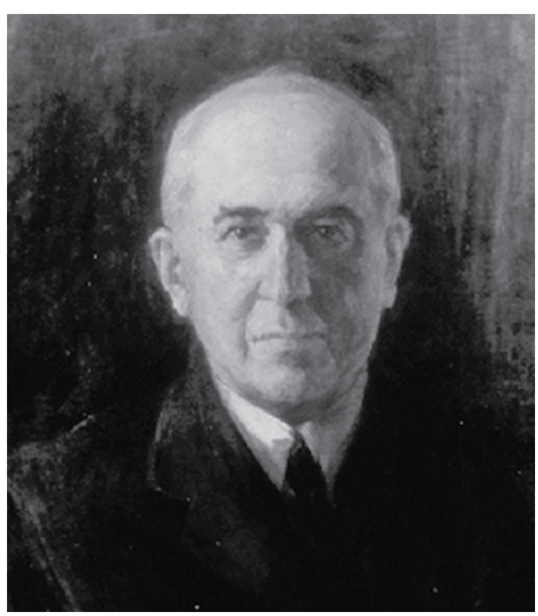
On interpreting deep sea data as evidence of Milankovitch cycles

Barry Lee Woolley

A phenomenological approach is used to suggest a nonlinear differential equation capable of replicating the period ratios in all data sets currently attributed to Milankovitch cycles. In particular, Duffing equation predictions are compared to the deep sea data set that J. D. Hayes *et al*. hypothesized as being due to Milankovitch cycles in the Quarternary. While not settling on one particular mechanism for the generation of this data set, mechanical analogues are invoked for illustrative purposes. These suggest a rapid, catastrophic event of global nature on a necessarily spinning earth as the cause of the observed effects.

In the immediate wake of Darwin's publication of his book postulating evolution, James Croll hypothesized that changes in the eccentricity of the earth's orbit would result in less sunlight reaching the earth, which in turn would cause ice ages.[1] That was in 1867. It wasn't until the middle of the next century that the quantitative aspects of this hypothesis were worked out by a Serbian astronomer named Milutin Milankovitch.[2] He came up with three sunlight cycles. These Milankovitch cycles result from the postulated variation in the amount of solar radiation reaching the earth's surface due to the changing eccentricity of the earth's orbit around the sun, axial tilt of the earth's rotational axis relative to its plane of orbit around the sun, and precession or wobble of this axial tilt. They are created by the gravitational forces of the planets and the moon acting on the earth. Extrapolating from astronomical observations, the longest of these cycles is calculated to have a period of 100,000 years. The other cycles have periods of approximately 40,000 and 20,000 years.[3]

Conjectured Milankovitch cycles were long ignored or dismissed as a serious contender for the cause of ice ages because they were believed to provide insufficient variation in solar radiation to effect such changes. For more than a hundred years after Croll put forth his hypothesis, there seemed to be a dearth of—or, rather, total absence of—physical evidence to support their existence. However, they were resurrected as being a possible cause in 1976 when a group of scientists used them to explain variations in the oxygen isotope ratio content in fossil shells taken from two bottom cores (combined into "one" for analytical purposes) drilled in a Southern Hemispheric ocean basin.[4]



Milutin Milankovitch (1879–1958) took James Croll's 1867 hypothesis (that ice ages were related to variation in the earth's orbit) and made it quantitative. The resulting Milankovitch cycles have had their own cycles of scientific acceptability.

The core was not dated by anything like layer counting. A few key dates were assigned to layers by various means including an examination of their fossil content, radiocarbon dating, and consideration of a magnetic reversal. This was used to establish the dates at major intervals, with the assumption that there was a constant (or linear) slow rate of deposition between these bench marks. Sedimentation surges were not considered.[5] Then the carbonate (shell) content was sampled along the core at regular intervals. After processing the samples by physical and chemical means, the ratios of oxygen isotopes were determined by mass spectroscopy. The variance of these as a function of position (or "time") along the core was subjected to standard signal processing Fourier analysis. Finally, comparison of the phases of the periods found in this data processing procedure as compared to those from tests done on radiolarians led to the insertion of time intervals (*viz.* 25,000 and 4,000 years) for consistency sake. A time interval of 60,000 years was hypothesized as being missing at the top of the core. A geologic time extent of approximately 450,000 years was investigated.

This paper will focus on the validity of the old earth interpretation of the deep-sea core Milankovitch cycle data. It will do so by observing that there might be a nonlinear differential equation for a non-astronomical physical phenomenon that would rapidly generate Milankovitch cycle data. The observation follows from a phenomenological approach to the topic.

Phenomenological approach

The interpretation of astronomical data has been controversial in the past. For example, even though there existed a cumbersome geometrical method devised by the Greeks to explain the motion of the planets, Johannes Kepler (1571–1630) persisted in trying to find mathematical relationships in the database built by Tycho Brahe's (1546–1601) observations of them. Kepler's laws or, more accurately, his data-determined mathematical relationships were then instrumental in Isaac Newton's (1643–1727) discovery of the laws of gravitation. Note that this major paradigm change from a burdensome explanation that could be made to work to a revolutionary simple one was brought about by first finding the mathematical relationships in the data set. So it has been for the particularly difficult problems of science.

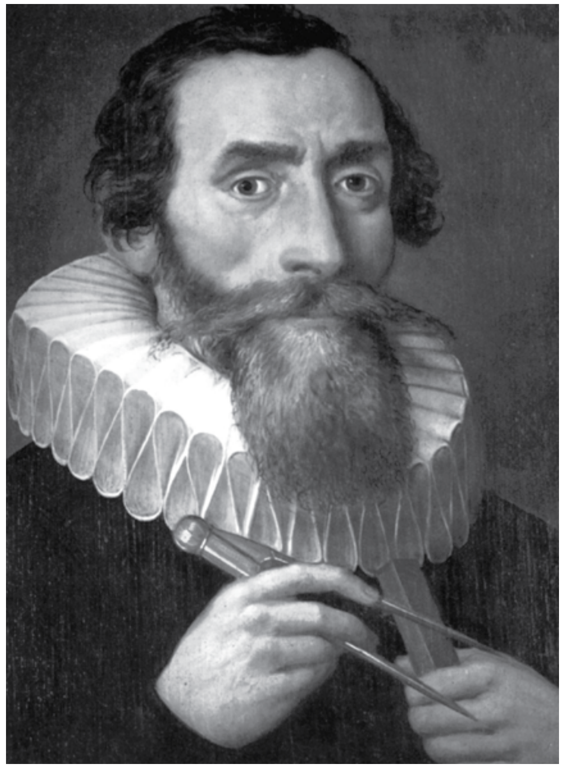


Image from <www.wikipedia.com>

Johannes Kepler used a phenomenological approach to devise his laws of planetary motion.

Even today some refractory problems of science continue to have scientists working on their solutions using this methodology.[6] This "phenomenological approach" seeks solely to mathematically describe the phenomenon under investigation, anticipating the future development of a theory corresponding to the mathematics. Hence a phenomenologist might look at the approximately 100,000, 40,000 and 20,000 year cycles purported to be due to Milankovitch cycles and ask if there weren't a differential equation whose solutions would generate period ratios of 1 to 2 to 5 (or 1/5 to 2/5 to 1, the case more likely to be observed).[7] Here is where the challenge lies.

Selecting the base equation

Differential equations are often used by scientists to describe and predict physical phenomena. A single form of differential equation may describe a specific phenomenon in a variety of contexts. For example, the wave equation is used to describe waves in the atmosphere, in the oceans, in the earth's interior—in most places where waves are found. Similarly, the so-called harmonic oscillator equation has application to the repetitive vibration of masses on springs, the small-amplitude motion of pendulums, the behavior of certain electrical circuits, etc. It applies to situations where oscillatory or repetitive motion is involved. A phenomenologist looking to describe a physical phenomenon would naturally take a look at whether or not a differential equation would be appropriate for the job. Of course he would have to look for one that was suitable to the physics and geometry suggested by his data set.[8]

If, for instance, it were known that a data set had a frequency associated with it, then it would be natural to consider both the wave equation and the harmonic oscillator equation as candidates to describe the phenomenon that produced it. The solution of physical or engineering problems involving either of these equations usually involves assuming a frequency dependence in the form of a harmonic function (i.e. either a sine or cosine function).[9]

If the wave equation were to have a frequency associated with it, it would come from either one of two potentially interrelated sources. It could come from the frequency associated with its propagation or it could come from the frequency of a resonance of this traveling wave between barriers. (There would have to be an integral number of half wavelengths of this wave being constrained between two wholly or partially reflecting barriers for a resonance to occur. This has to do with phase considerations for amplitude reinforcement.)

As a somewhat relevant illustration of this, consider two concentric large radius cylinders with a rib joining them. Striking either of the cylinders at a certain frequency will cause a propagating flexural wave in it with that frequency. When this excitation reaches the rib, it too will be set into motion.[10] This motion of the rib will become amplified into a resonance if the distance between the inner and outer cylinders (that is, the length of the rib) is an integral number of half wavelengths long for the flexural wave in the rib. That is, there will be a resonance of a traveling wave between two boundaries if its frequency is just right.

Knowing the resonance frequency in a data set, a phenomenologist would inquire about available wavespeeds in the medium associated with the set to determine what geometries would have to exist to generate the known resonance. Usually, however, there are a plethora of possibilities and the problem of perhaps missing the one that is relevant. So the use of a wave equation is likely to only serve as a reality check on the geometries associated with a resonance frequency.

A harmonic oscillator equation would also have a frequency associated with it. However, any repetitive wave that it describes would have to have a variable speed associated with it. During part of its cycle the wave would slow down, even momentarily stop, before it reversed its direction.[11] Physically this might be thought possible for a wave traveling in a medium between two barriers only if somehow there were a continuous and an overall drastic change in density in the medium through which it was traveling. (Elastic wavespeeds are often proportional to the inverse of the square root of the medium density.) Such conveniently extreme changes in density are rare, but it can be seen that any change in density would influence a modeler to consider dropping the simple use of the wave equation in favor of using the harmonic oscillator equation as a likely model for the physical situation.

Milankovitch cycles have *three* frequencies associated with them. Therefore a phenomenologist looking to describe Milankovitch cycles would have to look for an equation that was capable of having multiple frequencies associated with it. This clearly points to a nonlinear extension of the harmonic oscillator equation, a topic to be investigated after looking at the harmonic oscillator equation itself.

The forced harmonic oscillator equation

Isaac Newton's genius came up with the concept that force is equal to mass times acceleration. (His paradigm probing nature led him to conduct an experiment to see if the mass in this equation were the same as the mass postulated in his work on gravity.) In simple mathematical form, this can be written as F = ma. In the notation of differential equations for a simple one-dimensional problem this is $F = m\,d^2x/dt^2$, "x" being a distance measured from some reference point and "t" being time with the operator symbols indicating differential changes. Now consider a mass on a spring. If the mass is moved a distance x from its natural rest position, then the spring will exert a restoring force on it. That force may be proportional to the distance of its displacement, with the positive constant of proportionality symbolized as "k" (with a negative sign in front of it because it acts to restore the mass to its rest position). Hence $-kx = m\,d^2x/dt^2$ or, algebraically rearranging the equation, $m\,d^2x/dt^2 + kx = 0$. This is a linear equation and is easily solved by assuming x is proportional to a harmonic function. The equation is linear because it has x to the first power in it: it is called the harmonic oscillator equation.

However the restoring force may not simply be proportional to x. It may have additional terms like x^3, x^5, etc. in the expression that best describes it. (It will not have even powers of x in this expression because this would be unphysical: if the mass were above or below its rest position, the restoring force indicated by even powers would always be in the same direction rather than being a restoring force pushing up when the spring is extended below its rest position or pushing down when it is above this position.) In symbols the simplest extension of the above harmonic oscillator equation with an additional restoring force term is $m\,d^2x/dt^2 + kx - ax^3 = 0$. The negative sign in front of the positive constant "a" being there for valid physical reasons and results from the theory of equations. Note that now there is an x^3 or nonlinear term: this equation has no complete general harmonic solution. It is called the Duffing equation.

Now consider driving this harmonic oscillator by a force of amplitude "F" at an angular frequency of "ω". The equation becomes

$$m\frac{\partial^2\Phi}{\partial t^2} + D\frac{\partial\Phi}{\partial t} + k\Phi - a\,\Phi^3 = F\cos\omega t$$

(where the position has been indicated by "Φ" instead of the more restrictive notation of "x", and "cos" indicates the cosine function) which appears frequently in mechanics to describe not only masses on springs, but also such things as the forced motion of small amplitude pendulums, mechanical vibrations of clamped plates, etc. The relevance of this equation or the physical system it describes to the work here is that it displays responses with additional frequencies besides that of the forcing frequency. These are often fractions of the forcing frequency. (If the fractions are less than one, these are called subharmonics of the system. If the fractions are greater than one, they are called superharmonics of the physical system.) Indeed, just like the Milankovitch cycles, the forced Duffing equation displays prominent one half and one-fifth subharmonics. This is for most, if not all, problems of practical concern.[12] This makes it an ideal candidate for phenomenologically describing any physical system producing or exhibiting Milankovitch cycles.

Heuristic notations

It is possible that even a simple harmonic phenomenon could result in Duffing equation behavior (and, hence, give Milankovitch cycle results) if it were extensive enough. This can be illustrated by the following simple, yet perhaps unexpectedly relevant, example.

Consider a simple pendulum constituted by a mass "m" being swung back and forth, that is, from side to side, on a string of length "*l*". As usual, assume that the weight of the string is negligible compared to the mass at its end. Now have this pendulum swing in a plane. If the plane itself were to rotate at a steady angular velocity rate "ω," then the equation that it obeys would be the following:[13]

$$l \frac{\partial^2 \Theta}{\partial t^2} + g \sin\theta - l\,\omega^2 \sin\theta\cos\theta = \text{forcing function}$$

where "g" is the acceleration of gravity and "θ" is the angle from the rest position. Consulting the possible solutions of this equation in the position [theta] versus angular velocity space, it is found that for certain sets of forcing amplitudes on the mass and of angular accelerations, the solution for its motion will be an oscillation back and forth on either one of the two sides, with the mass never traversing through the rest position to the opposite side.

If the trigonometric functions in the above equation were to be expanded in their infinite series expansions, retaining only the most dominant terms up to the quadratic order, then the above equation would be transformed into the following more revealing and familiar one.

$$l \frac{\partial^2 \Theta}{\partial t^2} + (g - l\,\omega^2)\,\theta + [l\,\omega^2/2 - g/6]\,\theta^3 = \text{forcing function}$$

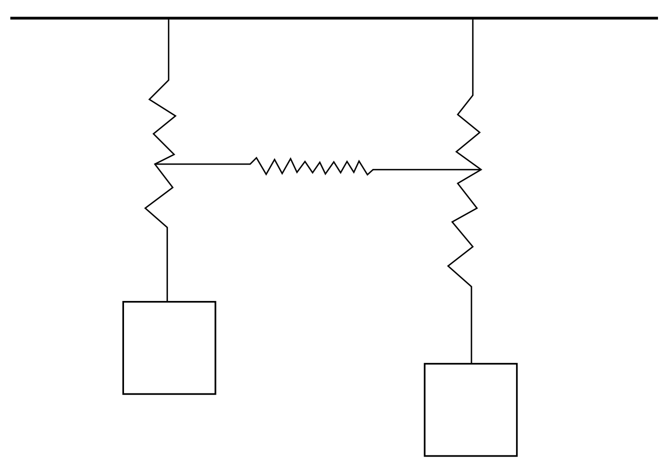
That is, an equation that could give Milankovitch cycle results. By analogy it is possible to conceive of a harmonic phenomenon constrained by the atmosphere (or lithosphere or bathysphere) on a spinning earth that would produce Milankovitch cycle results.

Now at this point it is premature and probably counterproductive (and contrary to a phenomenological approach) to proclaim a single specific widespread harmonic phenomenon on the rotating earth as the one that produced the reputed Milankovitch cycle data. And even though it is suspected that the oxygen isotope ratio data was produced in the ocean (or bathysphere), the harmonic source may not have been in it. It could have been in the atmosphere or in the lithosphere (i.e. Earth's rocky outer layer). It could have been some sort of physical resonance between atmospheric boundaries that may or may not exist today. It could have been a resonance in the lithosphere that was excited by convection currents deeper in the earth or by an extraterrestrial impact. It could have been a lot of things (e.g. things that directly affect the ocean core data like an unsuspected and undetected diagenetic effect, an uncompensated pressure-related habitat assumption, etc.), even some that have not yet been proposed.

Real data comparison

Consider the frequency data analysis that was presented in the 1976 seminal paper that spurred renewed interest in Milankovitch cycles. In it, two Fourier analysis values were found and combined to constitute the approximately 20,000 year short Milankovitch cycle. These values were 19,500 and 24,000 years. Two other Fourier analysis time intervals were taken into account to obtain the figure of approximately 40,000 years for the intermediate Milankovitch cycle. They were 42,000 and 43,000 years. Finally, two Fourier analysis derived numbers were associated with the approximately 100,000 year Milankovitch cycle. They were 106,000 and 400,000 year cycles. Note that 58% of the cyclicity of the oxygen isotope ratio data was attributed to the 106,000 year cycle, 27% to the 42,000 year cycle, and only 8% to the approximately 20,000 year cycle. This latter point about cyclicity was quite unexpected and disturbing: it was the opposite of what was originally calculated for the Milankovitch cycles, which had the strong cycle at about 20,000 years and the weakest cycle at 100,000 years.

Now if the data were produced by a phenomenon following an equation of the Duffing type, then it would be expected from the one fifth subharmonic that five times the average of the nearly 20,000 year cycle would come close to reproducing the 106,000 year cycle, which it does (being off by 2.6%). Now consider the one half frequency (or two fifths period) subharmonic. Two times the approximately 20,000 year average cycle is within 2.4% of the approximately 40,000 average year cycle. Furthermore, from the experience gained from the data analysis of the experimental work following the Duffing equation mentioned[12], data showing a period in the vicinity of 400,000 years comes as no surprise for a fundamental driving force with a period of 20,000 years. This one twentieth ratio was seen in the referenced experimental data set. So a Duffing equation interpretation of the deep-sea oxygen isotope ratio data does quite well when frequency ratios are examined.



Two *nearly* identical weakly coupled harmonic systems will generate oscillations of energy between them at the difference (and also the sum) of their frequencies.

Now what about relative magnitudes? If the data be interpreted in light of the author's past experience with Duffing equation data, then the relative magnitudes of the three periods in the data set are reversed (in exactly the same manner they would be in a Milankovitch cycle interpretation of the data set). A discussion of the physical interpretation of this is presented in the next section.

The existence of a 400,000 year period was just mentioned as being no surprise for a Duffing equation interpretation of the data. Its magnitude is expected to be at least equal in its influence on the data set to that of the 40,000 year one. That it isn't can probably be accounted for in large measure by the fact that the data set is only roughly 12% longer than this period.

A 60,000 year cycle corresponding to a one third subharmonic is sometimes seen; however it is usually much less pronounced than the least of the four other frequencies that have been discussed here.

Data enhancement and interpretation

In the previous section it was noted that there were two high frequency, short period resonances of 19,500 and 24,000 years in the oxygen isotope ratio data from the deep-sea cores. This might suggest there were two closely resonating and modally coupled physical systems (e.g. two ocean basins, two tectonic plates, both atmospheric hemispheres, etc.) that produced these period values. If it were assumed these were physically coupled, then the frequency of the modal coupling would be 104,000 years.[14] If this were really so, then the dominance of the long period Milankovitch cycle would not be such a mystery: the Fourier analysis would pick up this additional and additive modal coupling frequency effect.

It was also noted that there was both an unexpected relative prominence for the approximately 40,000 year Milankovitch cycle as well as a tight clustering (i.e. 42,000 and 43,000 years) of the two close Fourier analysis values that went into making up this number. It is conjectured that the breadth of the one half subharmonic of the Duffing equation is large enough to smear or merge the expected in-phase 39,000 and 48,000 year cycles into an enhanced 42,000 year peak.[15] This might help explain its prominence.

The insertion of time blocks in the 1976 seminal paper helped to enhance the Duffing equation interpretation of the period ratios in the data. So too, it is expected that any further signal processing (such as normalizing the data by a pre-Quartenary noise sample) will greatly strengthen the case put forth in this work; it is anticipated that this data enhancement will make frequency peaks sharper, frequency ratios more precise and closer to theoretical predictions, and the frequency spectrum closer to the theoretically predicted one. Furthermore, once the processing starts to take into account possible physical effects (like cut-off frequencies[16]), then it is anticipated that the complexity of the data's origin will tend toward a consensus interpretation.

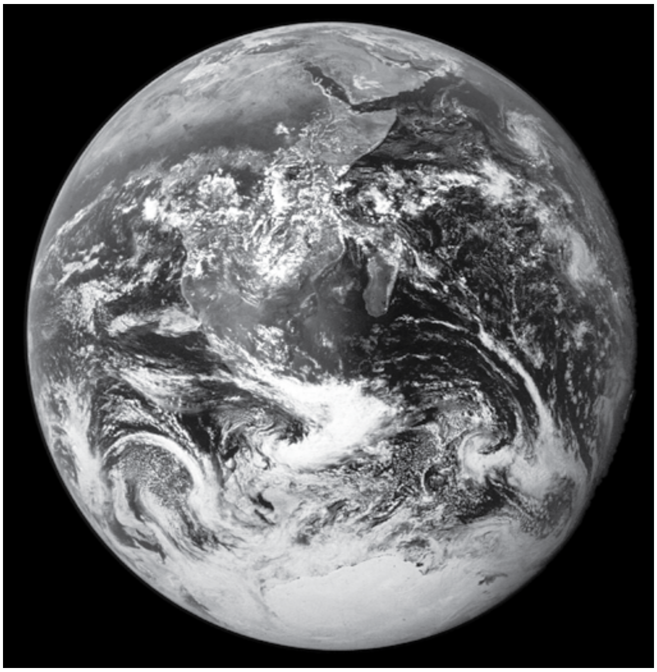
Summary and conclusions

The identification of variations in the oxygen isotope ratios (from shells extracted from deep-sea cores) with Milankovitch cycles has involved chronology date fixing, the insertion of large blocks of time units to bring about phase relation consistency, signal processing manipulations, less than realistic assumptions on sedimentation rates, and other activities. Some of these have spawned many critical comments and arguments. Surprise relations in the data among the magnitudes attributed to the effects of the different Milankovitch cycle components has led to subsequent uniformitarian attempts to justify these quantitative discrepancies between expectations and evidence. These give the appearance of being attempted *ad hoc* fixes of the problem. In addition to these environmental feedback mechanisms of deficient appeal, there is the difficulty in applying the Milankovitch cycle hypothesis to other physical situations where a supposedly global phenomenon only shows up in certain localities and not in others where it should also appear.[17] Somehow it appears that at least in some of the situations where a Milankovitch cycle hypothesis is invoked the physical explanation is incomplete or the mathematical tools applied to the problem are inadequate. Somehow it appears that a scientific paradigm has as its only recommendation the lack of a mathematical development sufficient to buttress a viable alternative. This paper seeks to correct that situation.

The deep-sea core Milankovitch cycle data appears to result from physical processes obeying the Duffing equation or an extension of it. Indeed, it has been conjectured herein that all physical effects generating data that can be taken as due to Milankovitch cycles can be obtained from other physical phenomena obeying the Duffing equation or an extension of it. It also appears that all physical processes and geometries that appear to be candidates for generating such data would occur in time envelopes that are probably several orders of magnitude less than that calculated by Milankovitch. (Looking at the wavespeeds and geometries available, forces the adoption of this general conclusion.)

The Duffing equation alternative put forth in this paper relies on a heuristic phenomenological approach guided by a historically-suggested paradigm. This reverse solution tack has the advantage of mathematical rigor at its onset. It has the reverse trait of all previous Milankovitch cycle work where existing physical evidence is given a theoretical basis.

Involved procedures for determining parameters for the nonlinear differential equation proffered in this paper, as well as related applications of this work to earth history, will be covered in future articles.



Most harmonic phenomena of significant extent on a necessarily spinning globe will follow the Duffing equation and generate seeming Milankovitch cycle data.

Possible future work and considerations

The most general pertinent differential equation encompassing all phenomena currently attributed to Milankovitch cycles would be similar to the following one:

$$m\frac{\partial^2\Phi}{\partial t^2} + D\frac{\partial\Phi}{\partial t} + k\Phi - a\Phi^3 + b\Phi^5 = F\cos\omega t + G\cos 2\omega t$$

The addition of the extra spring constant and forcing term was not necessarily a guess. The trigonometric identity

$$\cos^3 x = \frac{1}{4}(\cos 3x + 3\cos x)$$

first brought the one-third subharmonic to the attention of Duffing. (Cubing an assumed harmonic cosine function of time [as one would have to do in trying to solve the Duffing equation] would, due to the presence of the cosine term with three times the independent variable, automatically introduce a one third subharmonic in the solution. Here x would be taken as ωt.) The similar identity

$$\cos^5 x = \frac{1}{16}(\cos 5x + 5\cos 3x + 10\cos x)$$

would, when the cosine function was used as an assumed solution form in the above relevant equation, introduce a one fifth subharmonic in addition to the one third one and the fundamental frequency. Other related mathematical and physical considerations suggested the extra terms to the author.

A single mode analytical solution model will fail to replicate the full spectrum of solutions in physical or electrical analogue cases. The artificiality of using sinusoids to model waves, the increased frequency spectrum of finite signals, the possible couplings due to noise (or a dynamically evolving system), the mathematical reliance on trigonometric identities assuming only sinusoidal functions as a guide to differential equation development for models, etc., are all considerations that should be taken into account when comparing theory to reality. There are other physical complications that may also exist in a real physical situation, such as amplitude jumps[18] that have not been discussed in this paper which may appear in a real physical situation. These may account for the appearance of Milankovitch cycles in the Quarternary after their absence from the Cretaceous to the Tertiary.

References

1. Croll, J., On the eccentricity of the earth's orbit, and its physical relations to the Glacial Epoch, *Philosophical Magazine* **33**:119–131, 1867. Croll's first book, published in 1857, was entitled *The Philosophy of Theism*. He started a correspondence with Charles Darwin in 1864.
2. Milankovitch, M., *Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem* (Canon of Insolation of the Earth and Its Application to the Problem of the Ice Ages), Belgrade, Mihaila Curcica, 1941.
3. That is, the two smallest period cycles are roughly one-fifth and two-fifths of the cycle for eccentricity.
4. Hayes, J.D., Imbrie, J. and Shackleton, N.J., Variations in the earth's orbit: pacemaker of the Ice Ages, *Science* **194**:1121–1132, 1976. Besides oxygen isotope ratios taken from a particular species of planktonic foraminifera, Hayes *et al*. did an analysis on a particular radiolarian species and radiolarian assemblages. The isotope ratios were concluded to be indicative of the temperature of the seawater in which the minute foraminiferans lived before they died. This is hypothesized as being related to the extent of the earth's ice caps, which in turn is assumed to be directly related to the amount of sunlight striking the earth.
5 The dynamics in the 1976 paper were "fixed by assuming that the system is a time invariant linear system—that is, that its behavior in the time domain can be described by a linear differential equation with constant coefficients." Unphysical assumptions of linearity have proven useful in science and engineering in the past. For example, the assumption of linear stress through the cross section of an elastic plate with a wave in it definitely violates the known absence of shear at its boundaries, but has led to the development of the Timoshenko-Mindlin plate equation of motion that is widely used in the field of mechanical engineering. Minlin, R.D., Influence of rotary inertia and shear on flexural motions of isotopic, elastic plates, *Journal of Applied Mechanics* **18**:31–38, March 1951.
6. This methodology or approach is widely used in theoretical high-energy physics. Here the refractory problem is the attempt to find a fully satisfying scheme (analogous to the periodic table for the chemical elements) of the elementary particles that constitute matter.
7. N.B., the frequency ratios would be 1 to ½ to 1/5 for period ratios of 1 to 2 to 5, respectively. Frequency and periodicity are inversely related. If the period ratios were to be normalized by dividing each one by five, this would give period ratios of 1/5, 2/5 and 1, respectively.

8. Solutions to differential equations are often assumed to be of the form of certain functions best suited for the geometry of the problem under consideration. For instance, spherical harmonics for spherical geometries, Bessel functions for cylindrical geometries, and the harmonic functions (i.e. sines and cosines) for simple linear one- dimensional problems.

9. There is an intimate relationship between differential equations containing only even powers of their derivatives and harmonic functions which reproduce themselves when differentiated an even number of times.

10. An elastic medium can be a tensor one for waves in it. That is, it will have shear waves in it that can be polarized. A fluid medium, like water or air, will not sustain shear and hence will be a scalar one (only needing pressure and direction, not needing polarization, to describe waves in it). This paper has purposely presented an oversimplified view of the situation, leaving off such relevant considerations as symmetric waves, Goodier-Bishop decompositions of elastic waves, bulk waves in the rib, etc. Also the author has chosen not to talk about concentric spheres, a more pertinent example for an analogy to the earth's atmosphere, bathysphere (i.e. ocean), or lithosphere, but one for which a suitable data set was not available to him.

11. Actual reversal at the boundary may not be necessary. A global diagenetic effect of a physical or chemical nature, like the diffusion process that gives Liesegang rings, would have the slowing and stopping process produced chemically while progression took place in one direction. Henisch, H.K., *Crystals in Gels and Liesegang Rings: in Vitro Veritas* Cambridge University Press, New York, 1988. Creationists have put less emphasis on the diagenetic and dewatering effects of a global flood than they probably should have, despite superlative work in considering the ice age. Oard, M.J., *An Ice Age Caused by the Genesis Flood*, Institute for Creation Research, El Cajon, CA, p. 80, 1990.

12. Ueda found the one half subharmonic in 1985. See Ueda, Y., Random phenomena resulting from non-linearity in the system described by duffing's equation, *Int. J. Non-Linear Mechanics* **20**:481–491, 1985. The prominent one-fifth subharmonic was noted in 1989 from spectral analyses of acoustic returns excited by mechanical vibrations (definitely giving the numerous super and sub-harmonic roots attributed to the Duffing equation) investigated by the author. The object that was mechanically vibrating primarily consisted of a set of air-filled, capped concentric metal cylinders with periodic ribbing between them. It was rotated in water about an axis perpendicular to the cylinders' axes. This rotation axis was midway between the ends of the cylinders. Both cylinders had low thickness to radius ratios and were almost the same radius. The outer cylinder had a thickness on the order of one tenth of that of the inner one. The mechanical vibration was induced by a stress relieving 'pop'. Woolley, B.L., *Classification from Chaos*, Raytheon Submarine Signal Division, Portsmouth, RI, December 1989.

13. Note that positive solution existence would dictate that both

$$g > l\omega^2 \text{ and } g > l\omega^2\left(\frac{2! + 3!}{2!}\right) = 4\, l\, \omega^2 g$$

conditions that are always practically met for phenomena vibrating over a length that is small compared to the radius of the earth when the angular vibration is the earth's. However, note that the equation presented here is a suggestive, first approximation model. For example, g, the acceleration of gravity, should be modified by the effects of rotational distortion and centrifugal acceleration (see Turcotte, D.L. and Shubert, G., *Geodynamics: Applications of Continuum Physics to Geological Problems*, John Riley and Sons, New York, p. 204, 1982; formula 5-50.

14. If there were two *nearly* identical and weakly coupled harmonic oscillators (say two masses on separate springs with a weak spring connecting them), then energy would be transferred back and forth from one of them to the other. That is, each one would in turn have greater amplitude vibrations as the energy was transferred between them at a frequency rate equal to the difference (and the sum) of the natural frequencies of the two individual mass-spring systems. This is called modal coupling of the two nearly identical oscillators. Louisell, W.H., *Coupled Mode Theory and Parametric Electronics* John Wiley, New York, 1960.

15. Any real nonlinear phenomenon of finite duration will exhibit a range of amplitudes over time that will favor putting its energy into those of its resonances with the widest ranges as a function of frequency. There will in practice be a nearly equal partition of energy flux over all frequency intervals.

16. Junger, M.C. and Feit, D., *Sound, Structures, and Their Interaction*, Massachusetts Institute of Technology Press, Cambridge, MA, pp. 158–159, 1972.

17. Ruddiman, W.F. and McIntyre, A., Ocean mechanisms for amplification of the 23,000 year ice volume cycle, *Science* **212**:617–627, 1981. Ruddiman, W.F. and McIntyre, A., The mode and mechanism of the last deglaciation: oceanic evidence, *Quaternary Research* **16**:125–134, 1981. In the case of identifying Pleistocene carbonate sequences in the Caribbean basin with Milankovitch cycles, having the date-determining data differ by 15% in addition to having no equivalent deposition on the Florida and Bahamas platforms illustrates just how far earth scientists are willing to go to force-fit a theory to empirical facts.

18. Parzygnat, W.J. and Pao, Y.-H., Resonance phenomena in the non-linear vibration of plates governed by Duffing's Equation, *International Journal of Engineering Science* **16**:999–1017, 1978.

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