Appendix

Estimated error in this method

As derived in the paper, the following equation may be used to estimate the new period of a spectral peak after the original time domain signal corresponding to an original timescale T_0, has been stretched to fit into a larger timescale T_{new}:

$$P_{new} = P_0 \times \left(\frac{T_{new}}{T_0} \right) \quad \text{(A1)}$$

Since the timescales are assumed to be known exactly, the uncertainty in the new period estimate is

$$\Delta P_{new} = \left(\frac{T_{new}}{T_0} \right) \Delta P_0 \quad \text{(A2)}$$

where ΔP_0 is the uncertainty in the original period estimate. Here we now estimate that uncertainty.

This method assumed that the frequency of a spectral peak coincided *exactly* with one of the frequencies of the waves that had been superposed to obtain the original composite signal. However, this is often not the case: due to the finite number of frequencies comprising the spectrum, the tip of a spectral peak often falls *between* two consecutive frequencies. How much error results from this complication?

The maximum possible error in the frequency estimate of a spectral peak, due to this complication, is half the interval Δf between two consecutive discrete frequencies in the spectrum. This would be the case if the tip of the spectral peak fell exactly halfway between two of the discrete frequencies. We call the estimated (true) frequency of our spectral peak f_0. Since the period is just the reciprocal of the frequency, this implies that the original period estimate is

$$P_0 = \frac{1}{f_0} \quad \text{(A3)}$$

In this 'worst case' scenario, the two discrete frequencies that are closest to f_0 which actually appear in the power spectrum are $f_0 - \Delta f/2$ and $f_0 + \Delta f/2$. The longer of the two periods corresponding to these two frequencies is

$$P_{0,\text{larger}} = \frac{1}{f_0 - \frac{\Delta f}{2}} \quad \text{(A4)}$$

and the shorter of the two periods is:

$$P_{0,\text{smaller}} = \frac{1}{f_0 + \frac{\Delta f}{2}} \quad \text{(A5)}$$

Hence,

$$\Delta P_{0,\text{larger}} = P_{0,\text{larger}} - P_0 = \frac{1}{f_0 - \frac{\Delta f}{2}} - \frac{1}{f_0} = \frac{\frac{\Delta f}{2}}{f_0^2\left(1 - \frac{\Delta f}{2 f_0}\right)} \quad \text{(A6)}$$

Likewise,

$$\Delta P_{0,\text{smaller}} = P_0 - P_{0,\text{smaller}} = \frac{1}{f_0} - \frac{1}{f_0 + \frac{\Delta f}{2}} = \frac{\frac{\Delta f}{2}}{f_0^2(1 + \frac{\Delta f}{2 f_0})} \quad \text{(A7)}$$

There will not be much difference in the sizes of these two estimates, but they are not exactly the same. In order to simplify the analysis, we use the larger of these two error estimates, given by Equation (A6). Inserting Eq. (A6) into Eq. (A2) yields a formula for the maximum allowed error in our estimate of P_{new}:

$$\Delta P_{new} = \left(\frac{T_{new}}{T_0} \right) \left(\frac{1}{f_0} \frac{\frac{\Delta f}{2 f_0}}{(1 - \frac{\Delta f}{2 f_0})} \right) \quad \text{(A8)}$$

Or equivalently,

$$\Delta P_{new} = \left(\frac{T_{new}}{T_0} \right) \left(\frac{\Delta f}{2} \right) \frac{P_0^2}{(1 - P_0 \frac{\Delta f}{2})}. \quad \text{(A9)}$$

Note that we have implicitly assumed that the new frequency resolution is comparable to the original frequency resolution (i.e. the new value of Δf is roughly the same size as the original value of Δf, and preferably smaller), which was indeed true for this analysis (tables A1, A2, and A3). If, however, the new frequency resolution were noticeably coarser then the original frequency resolution, the uncertainty could be larger.

Note also that the functional form of the above equation implies that the (absolute) maximum allowed error ΔP_{new} will be much larger for the larger (~100 ka) periods (see tables A1, A2, and A3).

Of course, for a spectral peak that is closer than $\Delta f/2$ to the nearest discrete frequency, the actual allowed error will be less than this. However, we use Eq. (A9) as our error estimate, even though it is generally too large, for the following reasons. First, our uncertainty estimate ignored the fact that the amount of blurring of the spectra caused by the Blackman-Tukey (B-T) method was not the same in both the original and new calculations (reasons for this are explained in reference 12 in the paper). This can alter the shapes of the spectral peaks somewhat, thereby shifting slightly the frequency of prominent spectral peaks. Second, there was some additional error due to the Pacemaker authors' interpolation of the data when using the B–T method to obtain their original results (when obtaining my new results, I used very little or no interpolation). Because of this interpolation, the shape of the waveform corresponding to the original time T_0 was not *exactly* the same as the waveform corresponding to the new time T_{new}. For these reasons, this 'shortcut' method is only truly valid when both the original and new power spectra use exactly the same amount of interpolation from the original data *and* the same degree of spectral blurring. Because of these additional, unaccounted for, sources of error, we use Eq. (A9) as our expression for the uncertainty in the new period estimates, even though it generally overestimates somewhat the error due to a spectral peak not lying exactly halfway between two discrete frequencies. This gives us a bit of a 'buffer' for these other uncertainties, whose sizes are much harder to estimate. Tables A1 and A2 show that the difference between the two estimates for the new period, obtained both with this simple method and with the B–T method, was always less than or equal to the estimated error given by Eq. (A9). I did not attempt to calculate uncertainties for the new PATCH results, for reasons discussed in the text, but there was still good agreement (table A3) between the results obtained using the B-T method and the 'shortcut' presented in the text.

If one desires, one can ensure the same degree of blurring by imposing the condition that both the original and new power spectra have the same number of degrees of freedom, or equivalently, that the ratio m/n be the same before and after 'stretching' of the original timescale. When using the B-T method, the parameter n is the number of data points (after interpolation) and m is an integer (less than or equal to n) that determines the degree of blurring of the spectrum (the smaller the value of m, the greater the blurring). As an example, I have done this with the E49-18 $\delta^{18}O$ data. In the original Pacemaker paper, n was 122 and m was 50, yielding the ratio $m/n \approx 0.41$. For the new timescale, n was 107, so m was set to 44 to maintain this ratio. Figure A1 shows my replication of the original E49-18 $\delta^{18}O$ power

Table A1. Period estimates for the dominant spectral peaks calculated for RC11-120 summer sea surface temperature (SST), oxygen isotope values ($\delta^{18}O$), and percent abundance of the radiolarian species *Cyclodophora davisiana* (%C.d.). Original period estimates were reported to the nearest thousand years in reference 11, although I have here reported the smaller period estimates to one decimal place to reduce round-off error. The new period estimates were obtained using both the Blackman–Tukey method and the 'easy' method described in the text. Maximum allowed period uncertainties ΔP_{new} (reported to one significant figure) were obtained using the method described in this appendix.

RC11-120 SIMPLEX			
Original Time Interval (ka)	273.00		
New Time Interval (ka)	308.75		
Original Δf (cycles/ka)	0.001401		
New Δf (cycles/ka)	0.001293		
SST P_0 (ka)	102	37.6	21.0
SST P_{new}: easy method (ka)	115	42.5	23.8
SST P_{new}: B–T method (ka)	111	41.8	23.8
ΔP_{new} (ka)	9	1	0.4
$\delta^{18}O$ P_0 (ka)	95	37.6	23.8
$\delta^{18}O$ P_{new}: easy method (ka)	107	42.5	26.9
δ18O P_{new}: B–T method (ka)	111	43.0	27.1
ΔP_{new} (ka)	8	1	0.5
% C. d. P_0 (ka)	119	38.6	23.4
% C. d. P_{new}: easy method (ka)	135	43.7	26.5
% C. d. P_{new} : B–T method (ka)	129	43.0	26.7
ΔP_{new} (ka)	10	1	0.4

Table A2. Period estimates for the dominant spectral peaks calculated for E49-18 summer sea surface temperature (SST), oxygen isotope values ($\delta^{18}O$), and percent abundance of the radiolarian species *Cyclodophora davisiana* %C.d.). Original period estimates were reported to the nearest thousand years in reference 11, although I have here reported the smaller period estimates to one decimal place to reduce round-off error. The new period estimates were obtained using both the Blackman–Tukey method and the 'easy' method described in the text. Maximum allowed period uncertainties ΔP_{new} (reported to one significant figure) were obtained using the method described in this appendix.

E49-18 SIMPLEX			
Original Time Interval (ka)	363.0		
New Time Interval (ka)	402.8		
Original Δf (cycles/ka)	0.001119		
New Δf(cycles/ka)	0.001106		
SST P_0 (ka)	99	43.6	23.8*
SST P_{new}: easy method (ka)	110	48.4	26.4
SST P_{new}: B–T method (ka)	113	48.9	26.2
ΔP_{new} (ka)	6	1	0.4
$\delta^{18}O$ P_0 (ka)	105	47.1	25.2
$\delta^{18}O$ P_{new}: easy method (ka)	117	52.3	28.0
δ18O P_{new}: B–T method (ka)	120	53.2*	27.8
ΔP_{new} (ka)	7	1	0.4
% C. d. P_0 (ka)	149		
% C. d. P_{new}: easy method (ka)	165		
% C. d. P_{new} : B–T method (ka)	151		
ΔP_{new} (ka)	20		

spectrum, using the same number of frequencies (m+1) as the Pacemaker paper. Figure A1 compares favorably with the center chart (second row, second column) in figure 5 in the original Pacemaker paper. Likewise, figure A2 is the new power spectrum after the 'stretching' of the original timescale, but obtained with the same amount of spectral blurring as in figure A1 (m+1 frequencies were also used, although m in this case was 44 rather than 50). As expected, the shapes of the spectra are nearly identical.

As discussed in my original *ARJ* papers, I obtained estimates for the old and new periods by making high-resolution graphs of these low resolution spectra, since higher-resolution graphs makes it easier to estimate the frequencies of the prominent spectral peaks. Figure A3 is a high resolution (greater number of frequencies) version of figure A1, obtained as before with n = 122 and m = 50. Likewise, figure A4 is a high-resolution version of figure A2, obtained as before with n = 107 and m = 44. For both figures A3 and A4, the number of frequencies was set to 3m rather than m+1. Since in this particular case T_{new}/T_0 = 402.8 ka ÷ 363.0 ka ≈ 1.11, the new periods P_{new} are obtained by multiplying the period estimates shown in figure A3 by 1.11. As one can see from figure A4, there is *extremely* good agreement between the period estimates obtained using the 'shortcut' method and the period estimates obtained using the B-T method.

When re-doing the Pacemaker results after taking into account the age revision to the Brunhes-Matuyama magnetic reversal boundary, I attempted to be as charitable as possible to the Milankovitch theory in my choice of the parameter m,

Table A3. Period estimates for the dominant spectral peaks calculated for PATCH ELBOW summer sea surface temperature (SST), oxygen isotope ($\delta^{18}O$), and percent abundance of the radiolarian species *Cyclodophora davisiana* (%Cd). Original period estimates were reported to the nearest thousand years in reference 11, although I have here reported the smaller period estimates to one decimal place to reduce round-off error. The new period estimates were obtained using both the Blackman–Tukey method and the 'easy' method described in the text.

PATCH ELBOW				
Original Time Interval (ka)	486.0			
New Time Interval (ka)	543.6			
Original Δf (cycles/ka)	0.001119			
New Δf (cycles/ka)	0.000716			
SST P_0 (ka)	94	40.6	23.2	
SST P_{new}: easy method (ka)	105	45.4	25.9	
SST P_{new}: B–T method (ka)	112	45.8	26.4	
$\delta^{18}O$ P_0 (ka)	105	42.6	24.2	19.4
$\delta^{18}O$ P_{new}: easy method (ka)	117	47.6	27.1	21.7
δ18O P_{new}: B–T method (ka)	121	48.2	27.1	22.3
% C. d. P_0 (ka)	138	41.6	24.2	
% C. d. P_{new}: easy method (ka)	154	46.5	27.1	
% C. d. P_{new} : B–T method (ka)	155	45.8	26.9	

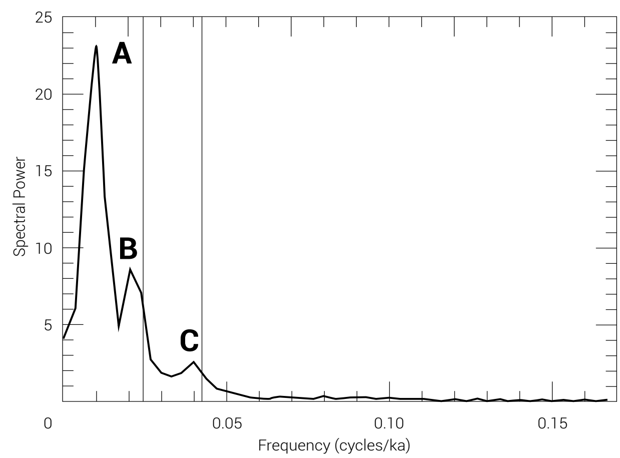

Figure A1. My replication of the E49-18 oxygen isotope ($\delta^{18}O$) power spectrum from the original Pacemaker paper. Vertical lines show the expected obliquity and precessional frequencies for the original time interval (due to the relative shortness of the time interval, the Pacemaker authors did not attempt to calculate an expected eccentricity frequency). Note that even before 'stretching' of the timescale, the agreement with Milankovitch expectations is not particularly good.

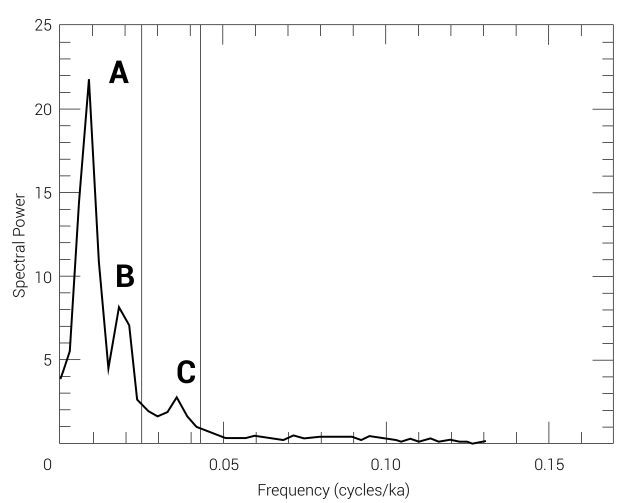

Figure A2. The E49-18 oxygen isotope ($\delta^{18}O$) power spectrum after the 'stretching' of the E49-18 timescale due to the revised age for the Brunhes-Matuyama magnetic reversal boundary, but with the same amount of spectral blurring as in figure A1. The expected obliquity and precessional frequencies (vertical lines) for the new time interval are almost exactly the same as for the old interval.

while still being reasonable. Hence I did not necessarily use the same degree of blurring as did the Pacemaker authors when re-doing the Pacemaker results. This different amount of spectral blurring is the primary reason for the poorer agreement between the new period estimates obtained using the two different methods in tables A1, A2, and A3. Nevertheless, the period estimates *still* agreed to within our estimated maximum allowed uncertainties, even with this source of error.

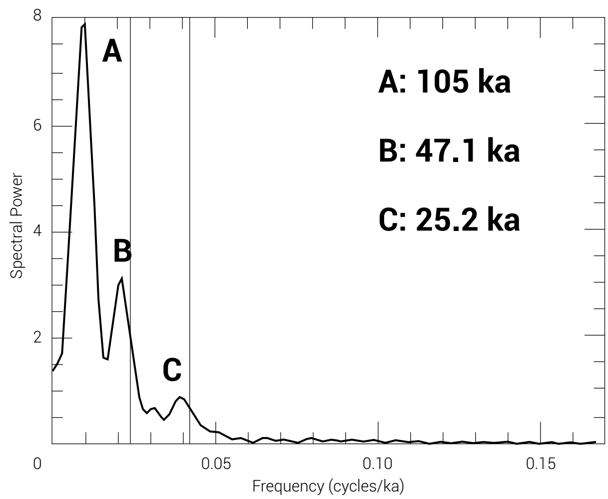

Figure A3. My high-resolution E49-18 oxygen isotope (δ[18]O) power spectrum for the original Pacemaker timescale, but subject to the same amount of spectral blurring as in figures A1 and A2. Numbers are my B-T period estimates (in ka) for the three prominent spectral peaks. Vertical lines show the expected obliquity and precessional frequencies for the original time interval.

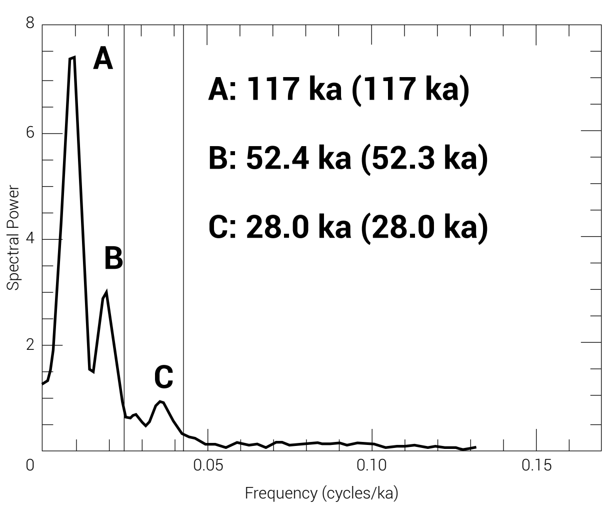

Figure A4. My high-resolution E49-18 oxygen isotope (δ[18]O) power spectrum after the 'stretching' of the E49-18 timescale due to the revised age for the Brunhes-Matuyama magnetic reversal boundary, but with the same amount of spectral blurring as in figures A1, A2, and A3. Numbers are my new Blackman-Tukey period estimates (in ka) for the three prominent spectral peaks. Numbers in parentheses are the new period estimates obtained by multiplying the old periods in Figure A3 by 1.11 (the 'shortcut' method). Vertical lines show the expected obliquity and precessional frequencies for the new time interval. Note the very good agreement between the period estimates obtained using the two different methods.

Of course, for someone making an internal critique of the Pacemaker paper, it is absolutely 'fair game' to use the same degree of spectral blurring as did the Pacemaker authors—one is under no logical obligation to alter the degree of blurring, as I did, in an attempt to be charitable to the original Pacemaker results. After all, this was the degree of blurring the Pacemaker authors *themselves* chose, so it is not at all unfair to maintain that same degree of blurring when re-doing the calculations. Doing so has the added bonus that it improves agreement between the period estimates obtained using the 'shortcut' and Blackman-Tukey methods.

Hence, for someone making an internal critique of the Pacemaker paper, the values of P_{new} obtained via the 'easy' method in tables A1, A2, and A3, are the estimated results using the reconstructed data sets, *my* chosen amount of spectral blurring, and the currently-accepted age estimate of 780 ka for the Brunhes-Matuyama magnetic reversal boundary. On the other hand, the values in figure A4 are the E49-18 values obtained using the reconstructed data sets, the Pacemaker authors' chosen amount of blurring, and the revised age for the Brunhes-Matuyama reversal boundary.

Of course, it is a simple matter to estimate *all* the new periods P_{new} that one would obtain using the Pacemaker authors' chosen degree of blurring. Simply multiply the reported 'Geologic' periods in their tables 3 and 4 by the appropriate 'stretch' factors: 1.13 for the RC11-120 periods, 1.11 for the E49-18 periods, and 1.12 for the PATCH periods (for the PATCH results, the 'unprewhitened' period estimates should be used). Hence even non-specialists can quickly see how the revised age of the Brunhes-Matuyama magnetic reversal boundary adversely affects these iconic results.

*Slight difference from print versions due to round-off error or slight error in reading numbers off my graphs. Though these errors are inconsequential, I wanted to correct them here for the sake of accuracy.